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Vol. 216 NOVEMBER, 1933 No. 5

# THE AMPLIFICATION AND RECORDING OF RAPID GEIGER-MÜLLER COUNTER IMPULSES.

BY

GORDON L. LOCHER, Ph.D.*

Bartol Research Foundation.

BARTOL RESEARCH FOUNDATION
Communication No. 77.

The application of Geiger-Müller counters to measurements of x-rays, faint light, and the counting of $\alpha$-particles has made it very desirable to develop an amplifier of very high sensitivity, for dealing with very feeble counter impulses, and recording apparatus for impulses at rapid rates.

These requirements are met by the circuit described here, which provides as much amplification as is consistent with the demands of stability, and allows the recording of impulses at rates of about 1400 per second, which is about ten times as fast as they are generated in any G-M counter so far described in print.

Figure 1 shows the circuit. Two stages of voltage amplification with high-gain tubes amplify the small negative impulses applied through $C_0$ and impress them on a pair of Western Electric Co. 256-A gas-filled tubes, whose circuit arrangement is identical with Wynn-Williams' "scale of two" recorder [1] which uses mercury-vapor thyratron tubes. Successive impulses start discharges in $T_1$ and $T_2$ alternately. Thus if the first impulse sets off $T_1$, the second sets off $T_2$

* National Research Fellow.

[1] Wynn-Williams, *Proc. Roy. Soc.*, A136, 312 (1932).

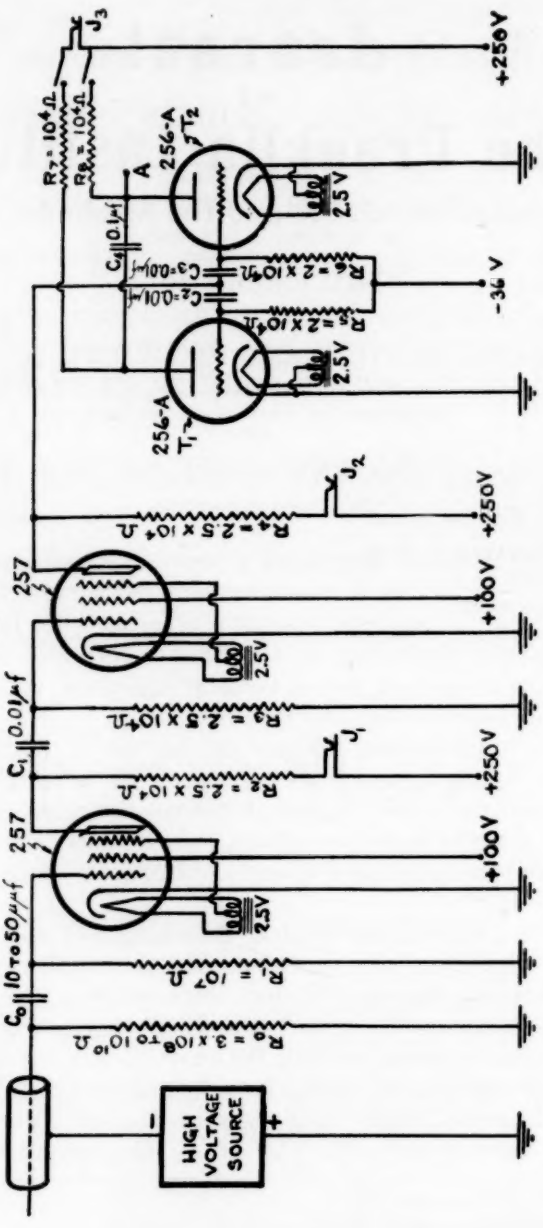

HIGH VOLTAGE SOURCE
+
−
$R_0$ = 3 x 10⁸ to 10¹⁰ Ω
$C_0$ 10 to 50 μμf
$R_1$ = 10⁷ Ω
257
2.5V
$C_1$ 0.01 μf
$R_2$ = 2.5 x 10⁴ Ω
$R_3$ = 2.5 x 10⁴ Ω
$R_4$ = 2.5 x 10⁴ Ω
$J_1$
$J_2$
$J_3$
+100V
+250V
$R_5$ = 2 x 10⁴ Ω
$R_6$ = 2 x 10⁴ Ω
$C_2$ = .001 μf
$C_3$ = .001 μf
$C_4$ 0.1 μf
$R_7$ = 10⁴ Ω
$R_8$ = 10⁴ Ω
A
256-A
$T_1$
$T_2$
-36 V

FIG. 1.

and extinguishes $T_1$, the third sets off $T_1$ and extinguishes $T_2$, and so on. A recording device introduced at $J_3$ thus counts half the impulses, $\pm$ 1 impulse. Any number of additional pairs of 256-A tubes may be used in tandem, in case the resolving time of the recording device is comparable with the mean time between the impulses. Each additional pair reduces the count by a factor of 2. The coupling condensers $C_2'$, $C_3'$ of the second stage are connected to $A$.

The maximum recording rate of the amplifier is determined by the cutoff time of the thyratron pair. Tests with an oscillograph showed a resolving time of considerably less than 0.0007 sec. It is important to know this constant, in order to calculate the number of impulses that arrive within intervals less than the resolving time, and are therefore not counted.

The plate current of the 256-A tube in the circuit shown is zero, when off, or 27 m.a., when on, for slow counting. It might be supposed that for rapid counting the average plate current would be directly proportional to the counting rate and could be used as a measure of the rate. This is not the case, since the "blocked" tube may not become completely de-ionized before the next impulse sets it off again.

#### CALCULATION OF THE NUMBER OF COUNTS LOST DUE TO GROUPING OF TWO OR MORE WITHIN THE RESOLVING TIME OF THE RECORDING APPARATUS.

The resolving time of an amplifier or recorder, for evenly-spaced impulses can approximately be determined with an oscillograph. The calculation below shows how many impulses are not counted due to doubles, triples, etc., arriving within intervals less than the resolving time, when the spacing is random.

Let $\tau$ = resolving time of apparatus (longest time),
$x$ = average no. impulses or events in $\tau$,
$T$ = time of observation.

The probability that $n$ events will occur in $\tau$, instead of $x$, is, according to Bateman,[2] $(x^n/n!)e^{-x}$. So the probability of none in $\tau$ is $e^{-x}$, of 1 in $\tau$, $xe^{-x}$, of 2 in $\tau$, $(x^2/2!)e^{-x}$ and so on.

[2] Rutherford, Geiger and Bateman, *Phil. Mag.*, **20**, 698 (1910).

The number of intervals $\tau$ with no events is $Ae^{-x}$; with 1, $Axe^{-x}$, and so on. It is easy to show that $A = T/\tau$. The number lost due to more than one falling within $\tau$ is

$$n = \frac{T}{\tau} e^{-x} \left( \frac{x^2}{2!} + \frac{2x^3}{3!} + \frac{3x^4}{4!} + \ldots \right),$$

or

$$n = \frac{T}{\tau} e^{-x} \left( x + \frac{x^2}{2!} + \frac{2x^3}{3!} + \ldots \right) - \frac{T}{\tau} x e^{-x}. \qquad (1)$$

Evaluating the series and substituting, we get

$$n = \frac{T}{\tau} (x - 1 + e^{-x}). \qquad (2)$$

The true number of events is

$$N = \frac{Tx}{\tau}.$$

So the fraction lost is

$$f = \frac{n}{N} = 1 - \frac{1}{x} + \frac{e^{-x}}{x}. \qquad (3)$$

Finally,

$$N_{(\text{true})} = N_{(\text{observed})} (x/1 - e^{-x}). \qquad (4)$$

Table I shows approximate values of $f$ for various values of $x$:

TABLE I.

| $x$ | $f$ (per cent.) |
|---|---|
| 1.0 | 36.8 |
| 0.75 | 29.6 |
| 0.5 | 21.4 |
| 0.2 | 9.5 |
| 0.1 | 5.0 |
| 0.05 | 2.0 |
| 0.01 | 1.0 |

**FACTORS LIMITING THE SPEED OF OPERATION OF GEIGER-MULLER COUNTERS.**

Besides the time constants of the amplifying circuit, four factors limit the speed of operation of Geiger-Müller counters, namely, the resolving time of the counting device, the time constant of the counter resistance and coupling condenser,

$R_0C_0$, the amplification factor of the amplifier, and the size and materials of the counter itself.

The first of these is perhaps the least serious. Mechanical counting devices have time constants of the order of 0.015 sec. or longer,[3] but their effective time constant may be reduced by 2, 4, 8, etc., by using 1, 2, 3, etc., "scale of two" thyratron pairs, as described above. An equally adequate but less convenient way is to use oscillographic recording.

The other three limitations are related to one another and cannot be treated separately. In general, the shortest resolving time is got by making the amplification constant of the amplifier as large as possible, $R_0$ and $C_0$ as small as possible, and the dimensions of the counting tube as small as possible. The limitation of the amplification factor is the same as that of any other high-gain amplifier, namely the tendency to instability, amplification of stray disturbances, and self-oscillation. The smallness of $C_0$ is determined by the amplification factor. The smallness of $R_0$ is determined by the size, materials, and gas pressure of the counter, since these determine its resistance during the intervals at which it is discharging.[4] During a discharge, if the resistance of the counter falls to too low a value, the discharge tends to maintain itself for a long period. To avoid this, $R_0$ cannot be less than a certain minimum value for a given counter. This minimum value depends on the amount of ionization in the counter, and on the materials of its construction.

The writer has lately found that photoelectric processes play an important part in the recovery time of counters. If the feeble light generated by the discharge contains wavelengths shorter than the photoelectric threshold of the metal of its cylinder-electrode, the discharge tends to spread throughout the entire tube, and to perpetuate itself. If, on the other hand, the photoelectric threshold of the metal is far in the ultraviolet (for example gold), and the gas emits little ultra-

[3] Locher, *Phys. Rev.*, **42**, 525 (1932).

[4] In a counter lately used by Locher and LeGalley for measurement of the absolute intensities of x-rays, the time constant was about 0.0038 sec., with $R_0 = 3.3 \times 10^8\ \Omega$, $C_0 = 25\ \mu\mu$f. The counter was 1 cm. in diameter, 1.5 cm. long, and was filled with Kr gas at 6 cm. pressure. Individual impulses as close together as 0.003 sec. were resolved. This work is in preparation for publication.

violet light, the recovery time is short. Thus an argon-filled counter has a shorter recovery time than one filled with air, krypton or hydrogen. On this basis, the order of probable preference of the several available gases should be: neon, xenon, argon, nitrogen, hydrogen, krypton, helium, oxygen, compounds such as $CO_2$, $CH_4$, and so on. An investigation is now in progress to determine quantitatively the relation of photoelectric phenomena and the recovery time of counters.

The writer is indebted to Dr. W. F. G. Swann for discussion of the statistical considerations, and to Mr. D. P. LeGalley for assistance in testing the circuit.

# THE DISTRIBUTION OF COSMIC RAY PATHS IN A VERTICAL CYLINDER.

BY

W. F. G. SWANN,

Director of the Bartol Research Foundation of The Franklin Institute.

BARTOL RESEARCH FOUNDATION
Communication No. 78.

A cylinder stands with its axis vertical. The problem is to find $f(l)$, the coefficient of $dl$ in the expression for the number of cosmic rays which travel distances between $l$ and $l + dl$ in the cylinder. The solution is of importance in connection with the detection of individual secondary cosmic rays by measurement of the ionization which they produce in such a cylinder, and in the interpretation of the results in terms of the ionization per centimeter of path of the rays. In this way it has been used by the writer in connection with experiments to be described in another place. For those whose interest is confined to the solution, and who are not interested in the derivation, it may be stated that the complete results are given in the section "Final Solution" at the end of the paper. They are accompanied by definitions of all the quantities concerned.[1]

*Assumptions and Definitions.* The problem will be discussed for the cases where the number of rays $\delta n$ passing per second through a horizontal element of area $ds$ within a solid angle $\sin\theta\, d\theta\, d\varphi$, whose axis is inclined at the angle $\theta$ to the vertical, is

$$\delta n = A \sin\theta \cos^3\theta\, d\theta\, d\varphi\, ds. \qquad (1)$$

Here, in accordance with empirical determinations, the intensity of the radiation is assumed to vary proportionally to $\cos^2\theta$ with angle from the vertical. The other $\cos\theta$ is, of course, to take account of the angle between the element of solid angle and the normal to $ds$.

[1] In an elaborate paper entitled "Höhenstrahlungskoinzidenzen in Zählröhren," *Berl. Ber.*, p. 830, 1931, Leo Tuwin considers the problem of cosmic ray paths in a cylinder. The paper does not appear, however, to have immediate application to the specific problem here solved.

We make the following definitions:

$L \equiv$ height of cylinder.

$a \equiv$ radius of cylinder

$$\lambda \equiv 2a/L; \qquad \xi \equiv 2a/l, \tag{2}$$

where $l$ is the variable length for which we seek $f(l)$.

$\delta n_{ss} \equiv$ contribution to $f(l)dl$ by the rays which enter and leave through the cylindrical surface.

$\delta n_{sb} \equiv$ contribution to $f(l)dl$ by the rays which enter through the side and leave through the base.

$\delta n_{st} \equiv$ contribution to $f(l)dl$ by the rays which enter through the top and leave through the side.

$\delta n_{tb} \equiv$ contribution to $f(l)dl$ by the rays which enter through the top and leave through the base.

*Cases Considered.* The mathematical derivations classify themselves according to the following cases:

Case 1. $2a < l$ and $l < L$, i.e., $\xi < 1$ and $\xi > \lambda$

Case 2. $2a < l$ and $l > L$, i.e., $\xi < 1$ and $\xi < \lambda$

Case 3. $l < 2a < L$, i.e., $\xi > 1$ and $\xi > \lambda$

Case 4. $l < L < 2a$, i.e., $\xi > 1$ and $\xi > \lambda$

Case 5. $L < l < 2a$, i.e., $\xi > 1$ and $\xi < \lambda$.

Not all the contributions $\delta n_{ss}$, $\delta n_{sb}$, $\delta n_{st}$, $\delta n_{tb}$ are represented in all of the cases 1–5. We have in fact, as follows:

Case 1. $$f(l)dl = \delta n_{ss_1} + \delta n_{sb_1} + \delta n_{st_1} \tag{3}$$

Case 2. $$f(l)dl = \delta n_{ss_2} + \delta n_{sb_2} + \delta n_{st_2} + \delta n_{tb_2} \tag{4}$$

Case 3. $$f(l)dl = \delta n_{ss_3} + \delta n_{sb_3} + \delta n_{st_3} \tag{5}$$

Case 4. $$f(l)dl = \delta n_{ss_4} + \delta n_{sb_4} + \delta n_{st_4} \tag{6}$$

Case 5. $$f(l)dl = \delta n_{ss_5} + \delta n_{sb_5} + \delta n_{st_5} + \delta n_{tb_5}, \tag{7}$$

where the *expressions* for the corresponding $\delta n$'s are not necessarily the same for all the cases, a fact which we have symbolized by the extra subscripts 1–5. Incidentally, as will presently appear, we always have

$$\delta n_{sb} = \delta n_{st}.$$

*Calculation of the* $\delta n_{ss}$. Consider rays which enter through an element $ds$ on the side, Fig. 1, $A$, and pass out through the side, and have a length between $l$ and $l + dl$. They pass out through the intersection of the *sphere*

$$r^2 = l^2 \tag{8}$$

with the cylinder

$$\begin{aligned} r^2 \sin^2 \theta &= a^2 + a^2 - 2a^2 \cos 2\varphi \\ &= 4a^2 \sin^2 \varphi. \end{aligned} \tag{9}$$

The intersection is given by

$$l \sin \theta = 2a \sin \varphi. \tag{10}$$

Keeping $\varphi$ constant, we see from (10) that the rays which lie between $l$ and $l + dl$ are related to those which lie between $\theta$ and $\theta + d\theta$ by

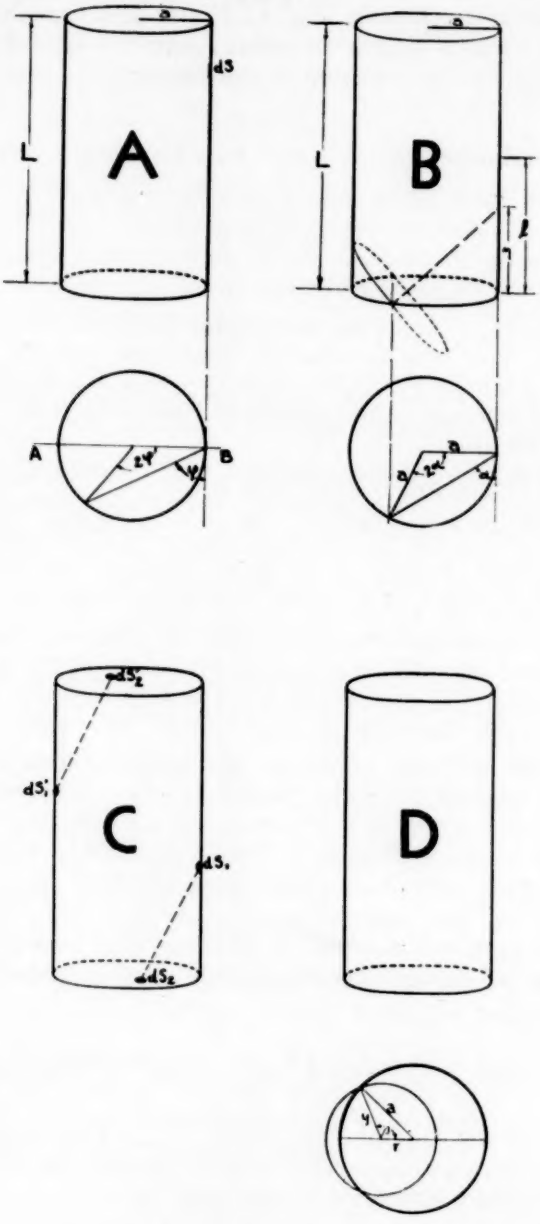
$$\begin{aligned} \sin \theta \, dl + l \cos \theta \, d\theta &= 0 \\ d\theta &= -\tan \theta \, dl/l. \end{aligned} \tag{11}$$

The number of rays entering $ds$ through the solid angle $\sin \theta \, d\theta \, d\varphi$ is, by (1), $A \sin^2 \theta \cos^2 \theta \sin \varphi \, d\theta \, d\varphi \, ds$. Hence, using (10) and (11) and ignoring the negative sign, which has no fundamental significance,[2] we have for the number of rays passing through $ds$ between $\varphi$ and $\varphi + d\varphi$, and having a length between $l$ and $l + dl$, the value $(A\xi^3 dl/l)$ $(1 - \xi^2 \sin^2 \varphi)^{1/2} \sin^4 \varphi \, d\varphi \, ds$. We have to integrate this expression appropriately for $ds$ and for $d\varphi$. We may replace $ds$ by $ad\psi d\eta$, where $\psi$ is measured in a plane parallel to the base and $d\eta$ is an element of length parallel to the length of the cylinder. We may then integrate from $\psi = 0$ to $2\pi$, and obtain the expression $(2\pi A a \xi^3 dl/l)$ $(1 - \xi^2 \sin^2 \varphi)^{1/2}$ $\sin^4 \varphi \, d\varphi \, d\eta$ which now has to be integrated over appropriate limits for $\varphi$, and for $\eta$ to obtain the desired value $\delta n_{ss}$. We thus obtain

$$\delta n_{ss} = 4\pi A a \xi^3 \frac{dl}{l} \int_{\eta_1}^{\eta_2} d\eta \int_{\varphi_1}^{\varphi_2} (1 - \xi^2 \sin^2 \varphi)^{1/2} \sin^4 \varphi \, d\varphi, \tag{12}$$

[2] Positive $d\theta$ corresponds to negative $dl$. A completely formal representation of the situation would be one in which $f(l)$ was defined as the coefficient of $|dl|$ in the number of rays lying between $l$ and $l + dl$.

FIG. 1.



in which we have taken account of the symmetry with respect to the diameter $AB$ by incorporating an extra 2, and in which $\varphi_1$ and $\varphi_2$ lie within the interval $0 - \pi/2$. $\varphi_1$ and $\varphi_2$ are in general functions of $\eta$, $a$, and $l$, and the limits $\eta_1$ and $\eta_2$ are functions of $l$, $a$ and $L$.

For case 1, the limits of $\varphi$ are 0 and $\pi/2$ for $\eta > l$. If $\eta < l$, however, while $\varphi_2 = \pi/2$, we have $\varphi_1 = \alpha$ as given, see Fig. 1, $B$, by

$$\eta^2 \tan^2 \theta = a^2 + a^2 - 2a^2 \cos 2\alpha = 4a^2 \sin^2 \alpha$$

which in view of (10) gives

$$\frac{\eta^2}{l^2} = 1 - \xi^2 \sin^2 \alpha. \tag{13}$$

The lower limit $\eta_1$ is given by

$$\eta_1^2 + (2a)^2 = l^2. \tag{14}$$

Thus

$$\delta n_{ss_1} = 4\pi A a \xi^3 \frac{dl}{l}\left[\int_l^L d\eta \int_0^{\pi/2} (1 - \xi^2 \sin^2 \varphi)^{1/2} \sin^4 \varphi \, d\varphi + \int_{(l^2-4a^2)^{1/2}}^{l} d\eta \int_\alpha^{\pi/2} (1 - \xi^2 \sin^2 \varphi)^{1/2} \sin^4 \varphi \, d\varphi\right]. \tag{15}$$

$\alpha$ is a function of $\eta$ through (13). It is convenient to change from variable $\eta$ to $\alpha$ through (13), so that

$$d\eta = -\frac{l\xi^2 \sin \alpha \cos \alpha \, d\alpha}{(1 - \xi^2 \sin^2 \alpha)^{1/2}}. \tag{16}$$

Equation (15) then becomes, on putting $L/l = \xi/\lambda$,

$$\delta n_{ss_1} = 4\pi A a \xi^3 dl\left[\left(\frac{\xi}{\lambda} - 1\right) \times \int_0^{\pi/2} (1 - \xi^2 \sin^2 \varphi)^{1/2} \sin^4 \varphi \, d\varphi + \xi^2 \int_0^{\pi/2} \frac{\sin \alpha \cos \alpha}{(1 - \xi^2 \sin^2 \alpha)^{1/2}} \times \int_\alpha^{\pi/2} (1 - \xi^2 \sin^2 \varphi)^{1/2} \sin^4 \varphi \, d\varphi\right]. \tag{17}$$

For case 2, since $l > L$, we have $\eta < l$ always; consequently, the first integral in (15) with respect to $\eta$ between the limits $l$ and $L$ is absent in the present case, and we are left with

$$\delta n_{ss_2} = \frac{4\pi A a \xi^3 dl}{l} \int_{(l^2-4a^2)^{1/2}}^{L} d\eta \times \int_{\alpha}^{\pi/2} (1 - \xi^2 \sin^2 \varphi)^{1/2} \sin^4 \varphi \, d\varphi, \tag{18}$$

where it will be observed that the upper limit of $\eta$ is now $L$. Again changing the variable from $\eta$ to $\alpha$, by means of (13), we find that the lower limit corresponds to $\alpha = \pi/2$ and the upper limit to $\alpha = \sin^{-1} [(1 - \xi^2/\lambda^2)^{1/2}/\xi]$, so that after reversing the limits to take account of the minus sign in (16), we have

$$\delta n_{ss_2} = 4\pi A a \xi^5 dl \int_{\alpha}^{\pi/2} \frac{\sin \alpha \cos \alpha \, d\alpha}{(1 - \xi^2 \sin^2 \alpha)^{1/2}} \times \int_{\alpha}^{\pi/2} (1 - \xi^2 \sin^2 \varphi)^{1/2} \sin^4 \varphi \, d\varphi, \tag{19}$$

where

$$\xi \sin \alpha_2 = (1 - \xi^2/\lambda^2)^{1/2}. \tag{20}$$

For case 3, we observe that since $\sin \theta$ cannot be greater than unity, the maximum value of $\varphi$ permitted by (10) is $\varphi = \sin^{-1} (1/\xi)$.

From $\eta = l$ to $\eta = L$, the limits of $\varphi$ are zero and $\sin^{-1} (1/\xi)$. The lower limit of $\eta$ is now zero; and from $\eta = 0$ to $\eta = l$, the limits of $\varphi$ are $\alpha$ as given by (13) and $\sin^{-1} (1/\xi)$. Thus, instead of the form (15), we now have for $\delta n_{ss}$

$$\delta n_{ss_3} = 4\pi A a \frac{\xi^3 dl}{l} \left[ \int_{l}^{L} d\eta \int_{0}^{\sin^{-1}(1/\xi)} \times (1 - \xi^2 \sin^2 \varphi)^{1/2} \sin^4 \varphi \, d\varphi + \int_{0}^{l} d\eta \times \int_{\alpha}^{\sin^{-1}(1/\xi)} (1 - \xi^2 \sin^2 \varphi)^{1/2} \sin^4 \varphi \, d\varphi \right]. \tag{21}$$

Integrating the first integral from $\eta = l$ to $L$, and changing the variable from $\eta$ to $\alpha$ in the second integral, we have

$$\delta n_{ss_3} = 4\pi A a \xi^3 dl \left[ \left( \frac{\xi}{\lambda} - 1 \right) \right.$$
$$\left. \times \int_0^{\sin^{-1}(1/\xi)} (1 - \xi^2 \sin^2 \varphi)^{1/2} \sin^4 \varphi \, d\varphi \right.$$
$$+ \xi^2 \int_0^{\sin^{-1}(1/\xi)} \frac{\sin \alpha \cos \alpha \, d\alpha}{(1 - \xi^2 \sin^2 \alpha)^{1/2}} \tag{22}$$
$$\times \int_\alpha^{\sin^{-1}(1/\xi)} (1 - \xi^2 \sin^2 \varphi)^{1/2} \sin^4 \varphi \, d\varphi \Big].$$

For case 4, we have exactly the same conditions as for case 3. Thus

$$\delta n_{ss_4} = 4\pi A a \xi^3 dl \left[ \left( \frac{\xi}{\lambda} - 1 \right) \right.$$
$$\times \int_0^{\sin^{-1}(1/\xi)} (1 - \xi^2 \sin^2 \varphi)^{1/2} \sin^4 \varphi \, d\varphi$$
$$+ \xi^2 \int_0^{\sin^{-1}(1/\xi)} \frac{\sin \alpha \cos \alpha \, d\alpha}{(1 - \xi^2 \sin^2 \alpha)^{1/2}} \tag{23}$$
$$\left. \times \int_\alpha^{\sin^{-1}(1/\xi)} (1 - \xi^2 \sin^2 \varphi)^{1/2} \sin^4 \varphi \, d\varphi \right].$$

For case 5, the form of the expression is the same as (18), except that the upper limit of $\varphi$ is $\sin^{-1}(1/\xi)$, and $\eta$ goes from 0 to $L$. Thus, as corresponding to (19) and (20), we get

$$\delta n_{ss} = 4\pi A a \xi^5 dl \int_{\alpha_2}^{\alpha_1} \frac{\sin \alpha \cos \alpha \, d\alpha}{(1 - \xi^2 \sin^2 \alpha)^{1/2}}$$
$$\times \int_\alpha^{(\sin^{-1} 1/\xi)} (1 - \xi^2 \sin^2 \varphi)^{1/2} \sin^4 \varphi \, d\varphi, \tag{24}$$

where

$$\left. \begin{aligned} \xi \sin \alpha_2 &= (1 - \xi^2/\lambda^2)^{1/2} \\ \xi \sin \alpha_1 &= 1. \end{aligned} \right\} \tag{25}$$

*Calculation of $\delta n_{sb}$ and $\delta n_{st}$.* We shall first show that $\delta n_{sb} = \delta n_{st}$.

Consider any contribution to $\delta n_{sb}$ from the entrance element $ds_1$, and the exit element $ds_2$, Fig. 1, *C*. The contribution is $A \cos^2 \theta \, d\omega_2 \, ds_1 \sin \varphi \sin \theta$ where $d\omega_2$ is the solid angle subtended at $ds_1$ by $ds_2$. The contribution in question

is, consequently, $A \cos^3 \theta \sin \theta \sin \varphi \, ds_1 \, ds_2/l^2$. This is the number of rays with lengths between $l$ and $l + dl$ passing through $ds_1$ within the solid angle $d\omega$, at the angles $\theta$, $\varphi$, determined by the direction $l$. Now, imagine all these rays reversed. The number with lengths between $l$ and $l + dl$ now entering $ds_2$ and passing within the solid angle $d\omega_1$ determined by $ds_1$ at the angle $\theta$, $\varphi$, is $A ds_2 \, d\omega_1 \cos^3 \theta$, i.e. $A \cos^3 \theta \sin \theta \sin \varphi \, ds_1 \, ds_2/l^2$, which is the same as the value found above. This quantity corresponding to entry of the rays through $ds_2$ and exit through $ds_1$ is, however, exactly equal to a contribution to $\delta n_{st}$ from the rays entering an element $ds_2'$ at the top and leaving through an element $ds_1'$ in the side, these two elements being numerically equal to $ds_2$ and $ds_1$, and being situated symmetrically to them in the sense indicated in Fig. 1, *C*. Thus to each contribution to $\delta n_{sb}$, there is a corresponding equal contribution to $\delta n_{st}$ and vice versa. Hence

$$\delta n_{sb} = \delta n_{st}. \tag{26}$$

We now proceed to the calculation of $\delta n_{sb}$ for the various cases concerned.

For a given value of $l$, contributions to $\delta n_{sb}$ are made by elements of the cylindrical surface whose $\eta$ is such that it is possible to draw from the element to the base at least one line whose length is $l$.

In general, the radius vector from an element of the cylindrical surface describes on the base an arc of a circle which subtends at $ds$ a cone whose angle $\theta$ is given by

$$l \cos \theta = \eta. \tag{27}$$

So that, for a given value of $\eta$,

$$d\theta = \frac{dl}{l} \cot \theta. \tag{28}$$

The number of rays from the element $ds = a d\psi d\eta$ falling within the solid angle $\sin \theta \, d\theta \, d\varphi$ is consequently, in view of (1), $A \sin^2 \theta \cos^2 \theta \sin \varphi \, d\theta \, d\varphi \, a d\psi d\eta$. Integrating with respect to $\psi$ and substituting for $d\theta$ from (28), we obtain the expression $2\pi A a (dl/l) \sin^2 \theta \cos^2 \theta \cot \theta \sin \varphi \, d\varphi \, d\eta$, or using (27) we obtain $2\pi A a (dl/l)(\eta/l)^3 (1 - \eta^2/l^2)^{1/2} d\eta \sin \varphi \, d\varphi$. To

obtain $\delta n_{sb}$, we must now integrate this expression between appropriate limits of $\varphi$ and $\eta$. There is symmetry as regards $\varphi$ in relation to any diameter such as $AB$, Fig. 1, $A$, so that

$$\delta n_{sb} = 4\pi A a \frac{dl}{l} \int_{\eta_1}^{\eta_2} \left(\frac{\eta}{l}\right)^3 \left(1 - \frac{\eta^2}{l^2}\right)^{1/2} d\eta \times \int_{\alpha}^{\pi/2} \sin \varphi \, d\varphi. \tag{29}$$

The lower limit for $\varphi$ is exactly the same as it was for all cases of the $\delta n_{ss}$, as indeed is obvious from the very nature of the geometry of the problem. The lower limit is, in fact, the quantity $\alpha$ defined by (13). The upper limit is always $\pi/2$ for $\delta n_{sb}$. For the $\delta n_{ss}$ it only has this value for cases 1 and 2. Integrating with respect to $\varphi$, and then changing the variable from $\eta$ to $\alpha$ by means of (13) and (16) we obtain

$$\delta n_{sb} = 4\pi A a \xi^3 dl \int_{\alpha_2}^{\alpha_1} (1 - \xi^2 \sin^2 \alpha) \sin^2 \alpha \cos^2 \alpha d\alpha,$$

in which $\alpha_1$ and $\alpha_2$ correspond through (13) to the lower and upper limits of $\eta$. The integral is easily evaluated, and we obtain

$$\delta n_{sb} = \frac{4\pi}{8} A a \xi^3 dl \left[ (1 - \xi^2/2)\alpha - \left(\frac{1 - \xi^2/2}{4}\right) \sin 4\alpha + \frac{\xi^2}{6} \sin^3 2\alpha \right]_{\alpha_2}^{\alpha_1}. \tag{30}$$

For case 1, $\eta_1 = (l^2 - 4a^2)^{1/2}$; $\eta_2 = l$, so that $\alpha_1 = \pi/2$; $\alpha_2 = 0$ and there results

$$\delta n_{sb_1} = \frac{4\pi}{16} A a \xi^3 (1 - \xi^2/2) \pi dl. \tag{31}$$

For case 2, $\eta_1 = (l^2 - 4a^2)^{1/2}$; $\eta_2 = L$, so that $\alpha_1 = \pi/2$; $\alpha_2 = \sin^{-1} (\sqrt{1 - \xi^2/\lambda^2}/\xi)$ and there results

$$\delta n_{sb_2} = \frac{4\pi}{8} A a \xi^3 \left[ (1 - \xi^2/2)(\pi/2 - \alpha_2) + \left(\frac{1 - \xi^2/2}{4}\right) \sin 4\alpha_2 - \frac{\xi^2}{6} \sin^3 2\alpha_2 \right] dl, \tag{32}$$

where $\alpha_2 = \sin^{-1}(\sqrt{1 - \xi^2/\lambda^2}/\xi)$; $0 < \alpha_2 < \pi/2$. For case 3, $\eta_1 = 0$; $\eta_2 = l$. Hence, $\alpha_1 = \sin^{-1}(1/\xi)$; $\alpha_2 = 0$ and there results

$$\delta n_{sb_3} = \frac{4\pi}{8} Aa\xi^3 \Big[ (1 - \xi^2/2)\alpha_1 - \left(\frac{1 - \xi^2/2}{4}\right) \sin 4\alpha_1 + \frac{\xi^2}{6} \sin^3 2\alpha_1 \Big] dl, \tag{33}$$

where $\alpha_1 = \sin^{-1}(1/\xi)$. For case 4, $\eta_1 = 0$; $\eta_2 = l$. Hence

$$\delta n_{sb_4} = \delta n_{sb_3}. \tag{34}$$

For case 5, $\eta_1 = 0$; $\eta_2 = L$. Hence

$$\alpha_1 = \sin^{-1}(1/\xi); \qquad \alpha_2 = \sin^{-1}(\sqrt{1 - \xi^2/\lambda^2}/\xi) \tag{35}$$

and there results

$$\delta n_{sb_5} = \frac{4\pi}{8} Aa\xi^3 \Big[ (1 - \xi^2/2)(\alpha_1 - \alpha_2) - \left(\frac{1 - \xi^2/2}{4}\right) (\sin 4\alpha_1 - \sin 4\alpha_2) + \frac{\xi^2}{6} (\sin^3 2\alpha_1 - \sin^3 2\alpha_2) \Big] dl, \tag{36}$$

where $\alpha_1$ and $\alpha_2$ have the values given by (35), with the understanding $0 < \alpha < \pi/2$.

*Calculation of the* $\delta n_{tb}$. Consider an element $ds = rd\psi dr$ on an annulus of radius $r$ on the top of the cylinder. Draw from $ds$ a line of length $l$, and let it sweep out a cone, whose base falls on the base of the cylinder and is a circle of radius $y$ given by

$$y^2 + L^2 = l^2.$$

This circle will fall inside the circle of radius $a$ if $y + r < a$, i.e., if

$$r < a - \sqrt{l^2 - L^2}. \tag{37}$$

We have

$$L/l = \cos\theta, \tag{38}$$

so that

$$d\theta = \cot\theta\, dl/l. \tag{39}$$

Using (1), we find for the number of rays passing through the element $rd\psi dr$ within the solid angle $\sin\theta\, d\theta\, d\varphi$, the value $A\cos^3\theta\sin\theta\, d\theta d\varphi\,(rd\psi dr)$. Integrating with respect to $\psi$ from 0 to $2\pi$ and substituting for $\theta$ and $d\theta$ from (38) and (39) we obtain $2\pi A(L/l)^4(dl/l)rdrd\varphi$. This has to be integrated over appropriate ranges of $r$ and $\varphi$ to give $\delta n_{tb}$. For $a - \sqrt{l^2 - L^2} > 0$ we have as follows: From $r = 0$ to $r = a - \sqrt{l^2 - L^2}$ the range of integration for $\varphi$ is $\varphi = 0$ to $2\pi$. For the remaining contribution we must first integrate with regard to $\varphi$ from $-\beta$ to $+\beta$, where $\beta$ is a function of $r$ determined by the intersection of the circle of radius $y$ with that of radius $a$ (see Fig. 1, $D$). Then we must integrate with respect to $r$ from $a - \sqrt{l^2 - L^2}$ to $a$. We thus obtain

$$\delta n_{tb} = 2\pi A\left(\frac{L}{l}\right)^4\frac{dl}{l}\left[\pi(a - \sqrt{l^2 - L^2})^2 + 2\int_{a-\sqrt{l^2-L^2}}^{a}\beta r dr\right] \tag{40}$$

for $a - \sqrt{l^2 - L^2} > 0$, where $\beta$ is determined by

$$\begin{aligned} a^2 &= y^2 + r^2 - 2yr\cos\beta \\ &= l^2 - L^2 + r^2 - 2r(l^2 - L^2)^{1/2}\cos\beta, \end{aligned}$$

so that

$$\cos\beta = \frac{l^2 - L^2 - a^2 + r^2}{2r(l^2 - L^2)^{1/2}} \tag{41}$$

with $0 < \beta < \pi$. Putting $2a/l \equiv \xi$ and $2a/L \equiv \lambda$, and putting $x \equiv 2r/l$, we obtain from (40) and (41), for $a - \sqrt{l^2 - L^2} > 0$,

$$\delta n_{tb} = \frac{4\pi Aa}{2\xi}\left(\frac{\xi}{\lambda}\right)^4\left[\frac{\pi}{2}\left(\xi - 2\sqrt{1 - \frac{\xi^2}{\lambda^2}}\right)^2 + \int_{(\xi-2\sqrt{1-\xi^2/\lambda^2})}^{\xi}\beta x dx\right]dl. \tag{42}$$

It is moreover easily seen that

$$\delta n_{tb} = \frac{4\pi A a}{2\xi}\left(\frac{\xi}{\lambda}\right)^4 dl \int_{2\sqrt{1-\xi^2/\lambda^2}-\xi}^{\xi} \beta x dx \tag{42A}$$

for $a - \sqrt{l^2 - L^2} < 0$. In both cases

$$\cos\beta = \frac{1}{4}\,\frac{4\left(1 - \frac{\xi^2}{\lambda^2}\right) + x^2 - \xi^2}{x\left(1 - \frac{\xi^2}{\lambda^2}\right)^{1/2}} \tag{43}$$

with $0 < \beta < \pi$. It may be noted also that

$$\delta n_{tb_2} = \delta n_{tb_1} = \delta n_{tb}. \tag{44}$$

*The Forms of the Integrals.* The indefinite integrals which have appeared in the foregoing are:

$$I_1 = \int (1 - \xi^2 \sin^2\varphi)^{1/2} \sin^4\varphi d\varphi \tag{45}$$

$$I_2 = \int \frac{\sin\alpha\cos\alpha}{(1 - \xi^2\sin^2\alpha)^{1/2}} \times \int_{\alpha}^{\pi/2} (1 - \xi^2\sin^2\varphi)^{1/2}\sin^4\varphi d\varphi, \text{ for } \xi < 1 \tag{46}$$

$$I_2^* = \int \frac{\sin\alpha\cos\alpha}{(1 - \xi^2\sin^2\alpha)^{1/2}} \times \int_{\alpha}^{\sin^{-1}(1/\xi)} (1 - \xi^2\sin^2\varphi)^{1/2}\sin^4\varphi d\varphi, \text{ for } \xi > 1 \tag{47}$$

$$I_3 = \int \beta x dx, \tag{48}$$

where $\beta$ has the value given by (43). The integral $I_1$ may be evaluated in terms of elliptic integrals. It will be sufficient to quote the result, whose truth may easily be verified by differentiation. We find

$$I_1 = \left(\frac{1-4\xi^2}{15\xi^2} - \frac{1}{5}\sin^2\varphi\right)\sin\varphi\cos\varphi\sqrt{1-\xi^2\sin^2\varphi} + \frac{2}{15}\frac{(1+2\xi^2)(1-\xi^2)}{\xi^4}f(\varphi,\xi) - \frac{(2+3\xi^2-8\xi^4)}{15\xi^4}E(\varphi,\xi), \tag{49}$$

where

$$E(\varphi,\xi) = \int_0^{\varphi}(1-\xi^2\sin^2\nu)^{1/2}d\nu$$

$$f(\varphi,\xi) = F(\varphi,\xi) \text{ if } \xi < 1$$

$$f(\varphi,\xi) = \frac{1}{\xi}F\left\{\sin^{-1}[\xi\sin\varphi], \frac{1}{\xi}\right\} \text{ if } \xi > 1,$$

and

$$F(\varphi,\xi) = \int_0^{\varphi}\frac{d\nu}{(1-\xi^2\sin^2\nu)^{1/2}}$$

and of course

$$F\left\{\sin^{-1}[\xi\sin\varphi], \frac{1}{\xi}\right\} = \int_0^{\sin^{-1}[\xi\sin\varphi]}\frac{d\nu}{\left(1-\frac{\sin^2\nu}{\xi^2}\right)^{1/2}}.$$

$E$ and $F$ are of course well known elliptic integrals which may be evaluated from tables.

The integral $I_2$ can be integrated by parts, and then becomes

$$I_2 = -\frac{1}{\xi^2}(1-\xi^2\sin^2\alpha)^{1/2} \times \int_a^{\pi/2}(1-\xi^2\sin^2\varphi)^{1/2}\sin^4\varphi d\varphi - \frac{1}{\xi^2}\int(1-\xi^2\sin^2\alpha)\sin^4\alpha d\alpha. \tag{50}$$

The second integral on the right hand side may be evaluated without difficulty, and the first integral is given by (49) evaluated between the proper limits and with $\xi < 1$. Thus, we find

$$I_2 = \frac{1}{\xi^2}\left(\frac{1-4\xi^2}{15\xi^2} - \frac{1}{5}\sin^2\alpha\right) \times (1-\xi^2\sin^2\alpha)\sin\alpha\cos\alpha$$
$$- \frac{2}{15}\frac{(1+2\xi^2)(1-\xi^2)}{\xi^6}\left\{F\left(\frac{\pi}{2},\xi\right) - F(\alpha,\xi)\right\}(1-\xi^2\sin^2\alpha)^{1/2} + \frac{2+3\xi^2-8\xi^4}{15\xi^6} \times \left\{E\left(\frac{\pi}{2},\xi\right) - E(\alpha,\xi)\right\}(1-\xi^2\sin^2\alpha)^{1/2}$$
$$- \left(\frac{3}{8} - \frac{5\xi^2}{16}\right)\frac{\alpha}{\xi^2} + \left(\frac{1}{4} - \frac{15}{64}\xi^2\right)\frac{\sin 2\alpha}{\xi^2} - \left(\frac{1}{32} - \frac{3\xi^2}{64}\right)\frac{\sin 4\alpha}{\xi^2} - \frac{\sin 6\alpha}{192}. \quad (51)$$

The integral $I_2^*$ is obtained in a similar way from (50) except that the first integral in (50) is now to be evaluated between $\alpha$ and $\sin^{-1}(1/\xi)$. Moreover $I_2^*$ is only desired for the case $\xi > l$. In this way we obtain

$$I_2^* = \frac{1}{\xi^2}\left(\frac{1-4\xi^2}{15\xi^2} - \frac{1}{5}\sin^2\alpha\right) \times (1-\xi^2\sin^2\alpha)^{1/2}\sin\alpha\cos\alpha$$
$$- \frac{2}{15}\frac{(1+2\xi^2)(1-\xi^2)}{\xi^7}\left\{F\left(\frac{\pi}{2}\cdot\frac{1}{\xi}\right) - F\left(\sin^{-1}[\xi\sin\alpha],\frac{1}{\xi}\right)\right\}(1-\xi^2\sin^2\alpha)^{1/2} + \frac{2+3\xi^2-8\xi^4}{15\xi^6}\left\{E(\sin^{-1}[1/\xi],\xi) - E(\alpha,\xi)\right\}(1-\xi^2\sin^2\alpha)^{1/2} - \left(\frac{3}{8} - \frac{5\xi^2}{16}\right)\frac{\alpha}{\xi^2}$$
$$+ \left(\frac{1}{4} - \frac{15\xi^2}{64}\right)\frac{\sin 2\alpha}{\xi^2} - \left(\frac{1}{32} - \frac{3\xi^2}{64}\right)\frac{\sin 4\alpha}{\xi^2} - \frac{\sin 6\alpha}{192}. \quad (52)$$

Turning now to $I_3$, we have a rather more intricate situation. Solving (43) for $x$, we find

$$x = 2\left(1 - \frac{\xi^2}{\lambda^2}\right)^{1/2}\left[\cos\beta \pm \left(\cos^2\beta - 1 + \frac{1}{k^2}\right)^{1/2}\right], \quad (53)$$

where

$$k \equiv 4(1 - \xi^2/\lambda^2)/\xi^2. \quad (54)$$

By an analysis of the geometry of the situation, we readily find as follows: The sign before the radical is positive if $\xi^2(1/\lambda^2 + 1/4) > 1$. For $\xi^2(1/\lambda^2 + 1/4) < 1$, $\beta$ ranges from zero, corresponding to the lower limit of $x$, to a value $\beta_2$, corresponding to the upper limit $x = \xi$. In doing so, $\beta$ passes through a maximum equal to $\sin^{-1} 1/k$ if $k < \sqrt{2}$, but does not *attain* a maximum if $k > \sqrt{2}$. In the former case the radical must be preceded by a negative sign for the range $\beta = 0$ to $\sin^{-1} 1/k$ and by a positive sign for the range $\beta = \sin^{-1} 1/k$ back to $\beta_2$. For the latter case the sign must be taken as negative throughout. With this understanding we shall carry both signs through until the final statement of the integrals. From (53)

$$x^2 = 4\left(1 - \frac{\xi^2}{\lambda^2}\right)\left[\cos 2\beta + \frac{1}{k^2} \pm \frac{2}{k}(1 - k^2\sin^2\beta)^{1/2}\cos\beta\right]. \quad (55)$$

Now

$$I_3 = \int \beta x dx = \frac{1}{2}\int \beta\left(\frac{dx^2}{d\beta}\right)d\beta = \frac{1}{2}\beta x^2 - \frac{1}{2}\int x^2 d\beta. \quad (56)$$

Inserting the values given by (55) and (56), we find

$$I_3 = \frac{1}{2}\beta x^2 - 2\left(1 - \frac{\xi^2}{\lambda^2}\right)\int\left[\cos 2\beta + \frac{1}{k^2} \pm \frac{2}{k}(1 - k^2\sin^2\beta)^{1/2}\cos\beta\right]d\beta. \quad (57)$$

On writing $x_1$ and $x_2$ for the limits of $x$, and $\beta_1$ and $\beta_2$ for the corresponding limits of $\beta$, we have, using (56),

$$|I_3|_x^{x_2} = \frac{1}{2}(x_2^2\beta_2 - x_1^2\beta_1) - \frac{k^2\xi^2}{2}\left[\frac{\sin 2\beta}{2} + \frac{\beta}{k^2}\right]_{\beta_1}^{\beta_2} - k\xi^2\left[\pm\int_{\beta_1}^{\beta_2}(1 - k^2\sin^2\beta)^{1/2}\cos\beta d\beta\right]. \quad (58)$$

We now have two cases to consider.

*Case A.* $a - \sqrt{l^2 - L^2} > 0$, i.e. $\xi^2(1/\lambda^2 + 1/4) > 1$. Here the limits for $x$ and the corresponding limits for $\beta$, as obtained from (43), are

$$x_1 = \xi - 2\sqrt{1 - \xi^2/\lambda^2};\ \cos\beta_1 = -1;\ \beta_1 = \pi;$$
$$x_2 = \xi;\ \cos\beta_2 = (1 - \xi^2/\lambda^2)^{1/2}/\xi = k/2.$$

In accordance with the foregoing remark, we use the plus sign for the whole range $\beta$ to $\pi$.

*Case B.* $a - \sqrt{l^2 - L^2} < 0$, i.e. $\xi^2(1/\lambda^2 + 1/4) < 1$. Here the limits for $x$ and the corresponding limits for $\beta$, as obtained from (43), are

$$x_1 = 2\sqrt{1 - \xi^2/\lambda^2} - \xi;\ \cos\beta_1 = 1;\ \beta_1 = 0;$$
$$x_2 = \xi;\ \cos\beta_2 = (1 - \xi^2/\lambda^2)^{1/2}/\xi = k/2.$$

Hence, bearing in mind the above remarks concerning the sign preceding the radical, we see that if $k < \sqrt{2}$, we must use the minus sign for the range 0 to $\sin^{-1}(1/k)$, and the plus sign for the range $\sin^{-1}(1/k)$ back to $\beta_2$. For $k > \sqrt{2}$ we must use the minus sign. Again, using (35), with the condition $0 < \alpha_2 < \pi/2$, we have, for both cases, $\sin\alpha_2 = \cos\beta_2$, and $\beta_2 = \pi/2 - \alpha_2$. Moreover, $\sin 2\beta_2 = \sin 2\alpha_2$. Inserting the foregoing limits in (58), and using the relations just cited, we therefore find for Case $A$:

$$|I|_{\xi-2\sqrt{1-\xi^2/\lambda^2}}^{\xi} = -\frac{\xi^2}{2}\left[\pi k(k-2) + \frac{k^2}{2}\sin 2\beta_2 + 2k\int_{\pi/2}^{\beta_2}(1 - k^2\sin^2\beta)^{1/2}\cos\beta\, d\beta + 2k\int_{\pi}^{\pi/2}(1 - k^2\sin^2\beta)\cos\beta\, d\beta\right].$$

Bearing in mind that the integral from 0 to $\pi/2$ is the negative

of the integral from $\pi/2$ to $\pi$, we thus have

$$|I|_{\xi-2\sqrt{1-\xi^2/\lambda^2}}^{\xi} = -\frac{\xi^2}{2}\Big[\pi k(k-2) + \frac{k^2}{2}\sin 2\beta_2 + 2k\int_0^{\beta_2}(1-k^2\sin^2\beta)^{1/2}\cos\beta\,d\beta\Big].$$

Corresponding to Case $B$, on the other hand, we have

$$\begin{aligned}|I_2|_{2\sqrt{1-\xi^2/\lambda^2}-\xi}^{\xi} &= -\frac{\xi^2}{2}\Big[\frac{k}{2}\sin 2\beta_2 - 2k \times\int_0^{\sin^{-1}(1/k)}(1-k^2\sin^2\beta)^{1/2}\cos\beta\,d\beta \\ &\quad + 2k\int_{\sin^{-1}(1/k)}^{\beta_2}(1-k^2\sin^2\beta)^{1/2}\cos\beta\,d\beta\Big] \\ &= -\frac{\xi^2}{2}\Big[\frac{k^2}{2}\sin 2\beta + 2k \times\int_0^{\beta_2}(1-k^2\sin^2\beta)^{1/2}\cos\beta\,d\beta \\ &\quad - 4k\int_0^{\sin^{-1}(1/k)}(1-k^2\sin^2\beta)^{1/2}\cos\beta\,d\beta\Big],\end{aligned}$$

if $k < \sqrt{2}$, and a corresponding expression with the last integral omitted and the second integral preceded by a negative sign for $k > \sqrt{2}$. Using Peirce's tables, p. 45, we find, for the indefinite integral,

$$\frac{\sin\beta}{2}\sqrt{1-k^2\sin^2\beta} + \frac{1}{2k}\sin^{-1}(k\sin\beta).$$

Inserting this value, utilizing the relation $\sin\alpha_2 = \cos\beta_2$, and observing that $\sqrt{1-k^2\sin^2\beta}$ is to be interpreted as positive (or zero) at all limits, and never negative, we find, for Case $A$, after some reductions:

$$|I|_{\xi-2\sqrt{1-\xi^2/\lambda^2}}^{\xi} = -\frac{\xi^2}{2}\left[4\pi(\sin\alpha_2 - 1)\sin\alpha_2 + \sin 2\alpha_2 + \sin^{-1}\sin 2\alpha_2\right] \tag{59}$$

for $\xi^2(1/\lambda^2 + 1/4) > 1$, where $\sin^{-1} \sin 2\alpha_2$ is to be interpreted as an angle whose sine is $2\alpha_2$ and which is $< \pi/2$. Incidentally it may be noted that since we always have $\alpha_2 < \pi/6$ for Case 1, $\sin^{-1} \sin 2\alpha_2 = 2\alpha_2$. For Case $B$, observing that cases $k < \sqrt{2}$ and $k > \sqrt{2}$ correspond respectively to $\alpha_2 < \pi/4$ and $\alpha_2 > \pi/4$, we find

$$|I|^{\xi}_{2\sqrt{1-\xi^2/\lambda^2}-\xi} = -\frac{\xi^2}{2}[\sin 2\alpha_2 + \sin^{-1} \sin 2\alpha_2 - \pi]$$

for $\alpha_2 < \pi/4$ and $\xi^2(1/\lambda^2 + 1/4) < 1$, and

$$|I|^{\xi}_{2\sqrt{1-\xi^2/\lambda^2}-\xi} = -\frac{\xi^2}{2}[\sin 2\alpha_2 - \sin^{-1} \sin 2\alpha_2]$$

for $\alpha_2 > \pi/4$ and $\xi^2(1/\lambda^2 + 1/4) < 1$.

In these formulæ $\sin^{-1} \sin 2\alpha_2$ is to be interpreted as an angle $< \pi/2$. If, however, we omit this reservation, both expressions collapse to

$$|I|^{\xi}_{2\sqrt{1-\xi^2/\lambda^2}-\xi} = -\frac{\xi^2}{2}[\sin 2\alpha_2 + 2\alpha_2 - \pi] \qquad (60)$$

for all ranges of $\alpha_2$ and for $\xi^2(1/\lambda^2 + 1/4) < 1$.

*Completion of Calculation of* $\delta n_{tb}$. Having obtained (59) and (60) we are now in a position to write down $\delta n_{tb}$ explicitly from (42) and (42A). All cases collapse to a single expression,

$$\delta n_{tb} = 4\pi A a \left(\frac{\xi}{\lambda}\right)^4 \frac{\xi}{4}[\pi - \sin 2\alpha_2 - 2\alpha_2]dl. \qquad (61)$$

This expression is applicable to Cases $A$ and $B$ and for all the existing ranges of $\alpha_2$.

*Collection of Material.* We are now in possession of the various elements which go to make up $f(l)$ for the various cases, and it remains to piece them together. We have

*Case 1.* $f(l)$ is given by (3). $\delta n_{ss_1}$ is given by (17) with the limits 0 and $\pi/2$ inserted in $I_1$ and $I_2$ (see equations (45) and (46)) as given by (49) and (51). $\delta n_{sb_1}$ is given explicitly by (31) and $\delta n_{st_1}$ is equal to it.

*Case 2.* $f(l)$ is given by (4). $\delta n_{ss_2}$ is given by (19) by inserting the $\alpha_2$ as given by (20) in the integral, which

is the integral $I_2$ (see (46)), as explicitly given by (51). $\delta n_{sb_2}$ is given explicitly by (32), and $\delta n_{st_2}$ is equal to it. $\delta n_{tb_2} = \delta n_{tb}$, and is given by (61) and (62).

*Case 3.* $f(l)$ is given by (5). $\delta n_{ss_3}$ is given by (22) by inserting the appropriate limits in (45) and (47) as given explicitly by (49) and (52). $\delta n_{sb_3}$ is given by (33), and $\delta n_{st_3}$ is equal to it.

*Case 4.* $f(l)$ is given by (6). $\delta n_{ss_4}$ is given by (23) and is exactly equal to $\delta n_{ss_3}$. $\delta n_{sb_4}$ is, by (34), equal to $\delta n_{sb_3}$ and is given by (33). A similar remark applies to $\delta n_{st_4}$.

*Case 5.* $f(l)$ is given by (7). $\delta n_{ss_5}$ is given by (24) by inserting the $\alpha_1$ given by (25) in the integral, which is the integral $I_2^*$ of (47) as given explicitly by (52). $\delta n_{sb_5}$ is given explicitly by (36) and $\delta n_{st_5}$ is equal to it. $\delta n_{tb_5} = \delta n_{tb}$ and is given by (61) and (62).

### FINAL SOLUTION.

In accordance with the policy stated at the beginning of this paper, we shall now restate all the definitions and assumptions made, and shall write down the final expression for $f(l)$ in accordance with the principles of the last section.

*Fundamental Assumptions.* The number of rays $\delta n$ passing per second through a horizontal element of area $ds$ within a solid angle $d\omega$, whose axis is inclined at an angle $\theta$ to the vertical, is

$$\delta n = A \cos^3 \theta ds \, d\omega,$$

where $A$ is a constant.

*Definitions.* Height of cylinder $\equiv L$; radius of cylinder $\equiv a$; $\lambda \equiv 2a/L$. $f(l)dl$ = number of rays passing through cylinder with lengths lying between $l$ and $l + dl$. $\xi \equiv 2a/l$. In the expressions which follow, $\alpha_1 = \sin^{-1}(1/\xi)$; $\alpha_2 = \sin^{-1}(\sqrt{1 - \xi^2/\lambda^2}/\xi)$; $0 < \alpha_1 < \pi/2$; $0 < \alpha_2 < \pi/2$

$$F(\alpha, \xi) \equiv \int_0^{\alpha} \frac{dv}{(1 - \xi^2 \sin^2 v)^{1/2}}, \qquad \text{for } \xi < 1$$

$$E(\alpha, \xi) \equiv \int_0^{\alpha} (1 - \xi^2 \sin^2 v)^{1/2} dv.$$

*Final expressions for $L > 2a$, i.e. $\lambda < 1$, which case utilizes expressions from cases 1, 2, and 3.*

$$f(l) = 4\pi Aa\left[\frac{1}{15\lambda}\left\{2(1+2\xi^2)(1-\xi^2)F\left(\frac{\pi}{2},\xi\right) - (2+3\xi^2-8\xi^4)E\left(\frac{\pi}{2},\xi\right)\right\} + \frac{\pi\xi^3}{16}\left(\frac{3}{2}\xi^2-1\right)\right] \tag{62}$$

for $\lambda < \xi < 1$.

$$f(l) = 4\pi Aa\left[\frac{1}{15\lambda}\left\{2(1+2\xi^2)(1-\xi^2)\left[F\left(\frac{\pi}{2},\xi\right) - F(\alpha_2,\xi)\right] - (2+3\xi^2-8\xi^4)\left[E\left(\frac{\pi}{2},\xi\right) - E(\alpha_2,\xi)\right]\right\} + \frac{\xi^3}{8}\left\{\left(\frac{2\xi^2}{\lambda^4}+\frac{3\xi^2}{4}-\frac{1}{2}\right)\times(\pi-2\alpha_2) - \left(\frac{14\xi^2}{5\lambda^4}-\frac{16\xi^2}{15\lambda^2}-\frac{15\xi^2}{8} - \frac{8}{15\lambda^2}+2\right)\sin 2\alpha_2 + \left(\frac{3}{4}-\frac{5\xi^2}{8}\right)\sin 4\alpha_2 + \frac{\xi^2}{24}\sin 6\alpha_2 - \frac{\xi^2}{3}\sin^3 2\alpha_2\right\}\right] \tag{63}$$

for $\xi < \lambda < 1$.

$$f(l) = 4\pi Aa\left[\frac{1}{15\lambda}\left\{\frac{2(1+2\xi^2)(1-\xi^2)}{\xi}F\left(\frac{\pi}{2},\frac{1}{\xi}\right) - (2+3\xi^2-8\xi^4)E(\alpha_1,\xi)\right\} + \frac{\xi^2}{8}\left\{\left(\frac{3}{2}\xi^2-1\right)\alpha_1 + \left(2-\frac{15}{8}\xi^2\right)\sin 2\alpha_1 - \left(\frac{3}{4}-\frac{5}{8}\xi^2\right)\sin 4\alpha_1 - \frac{\xi^2}{24}\sin 6\alpha_1 + \frac{\xi^2}{3}\sin^3 2\alpha_1\right\}\right] \tag{64}$$

for $\xi > 1 > \lambda$.

*Final expressions for $L < 2a$, i.e. $\lambda > 1$, which utilizes expressions for cases 2, 4, and 5.*

$$f(l) = \text{same expression as (63) for } \xi < 1 < \lambda$$

$$f(l) = \text{same expression as (64) for } \xi > \lambda > 1$$

since $\delta n_{ss_3} = \delta n_{ss_4}$, and $\delta n_{sb_3} = \delta n_{sb_4} = \delta n_{st_3} = \delta n_{st_4}$.

$$\begin{aligned} f(l) = 4\pi A a \Bigg[ & \frac{1}{15\lambda} \Bigg\{ \frac{2(1+2\xi^2)(1-\xi^2)}{\xi} \Bigg[ F\left(\frac{\pi}{2}, \frac{1}{\xi}\right) \\ & - F\left(\sin^{-1}\sqrt{1-\xi^2/\lambda^2}, \frac{1}{\xi}\right)\Bigg] \\ & - (2+3\xi^2-8\xi^4)[E(\sin^{-1} 1/\xi, \xi) - E(\alpha_2, \xi)]\Bigg\} \\ & + \frac{\xi^3}{8}\Bigg\{\left(\frac{3}{2}\xi^2 - 1\right)(\alpha_1 - \alpha_2) \\ & + \left(2 - \frac{15\xi^2}{8}\right)(\sin 2\alpha_1 - \sin 2\alpha_2) \\ & - \left(\frac{3}{4} - \frac{5}{8}\xi^2\right)(\sin 4\alpha_1 - \sin 4\alpha_2) \\ & - \frac{\xi^2}{24}(\sin 6\alpha_1 - \sin 6\alpha_2) \\ & + \frac{\xi^2}{3}(\sin^3 2\alpha_1 - \sin^3 2\alpha_2)\Bigg\} \\ & + \frac{\xi^3}{4}\Bigg\{\frac{\xi^2}{\lambda^4}(\pi - 2\alpha_2) - \Bigg(\frac{7\xi^2}{5\lambda^4} - \frac{8\xi^2}{15\lambda^2} \\ & \qquad - \frac{4}{15\lambda^2}\Bigg)\sin 2\alpha_2\Bigg\}\Bigg] \end{aligned} \tag{65}$$

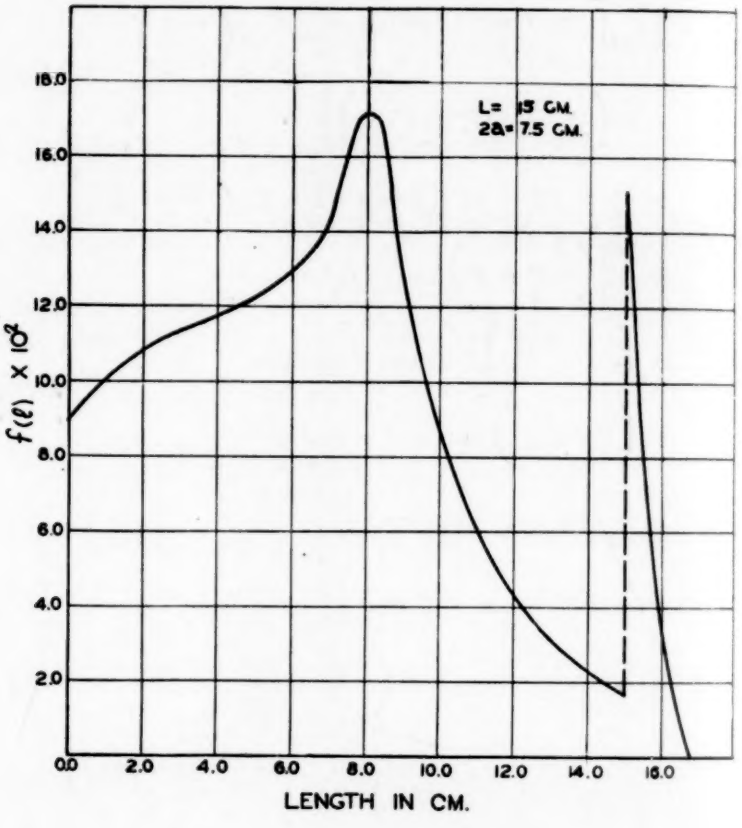
for $\lambda > \xi > 1$.

In all cases, of course, $l$ is limited by the condition $l^2 < (2a)^2 + L^2$, i.e., by the condition $(1 + 1/\lambda^2)^{1/2} > 1/\xi$.

Figure 2, *A*, gives an example of $f(l)$ for a cylinder for which $2a = 7.5$ cms. and $L = 4a = 15$ cms. Figure 2, *B*, gives a case where $2a = 15$ cms. and $L = 2a/2 = 7.5$ cms.

The curves obviously show interesting features in the vicinity of $l = L$. Figure 2, *A*, also shows an interesting maximum near $l = 2a$. Some of these features will, however, be discussed in another place in connection with their significance in certain experiments on the ionization per centimeter of

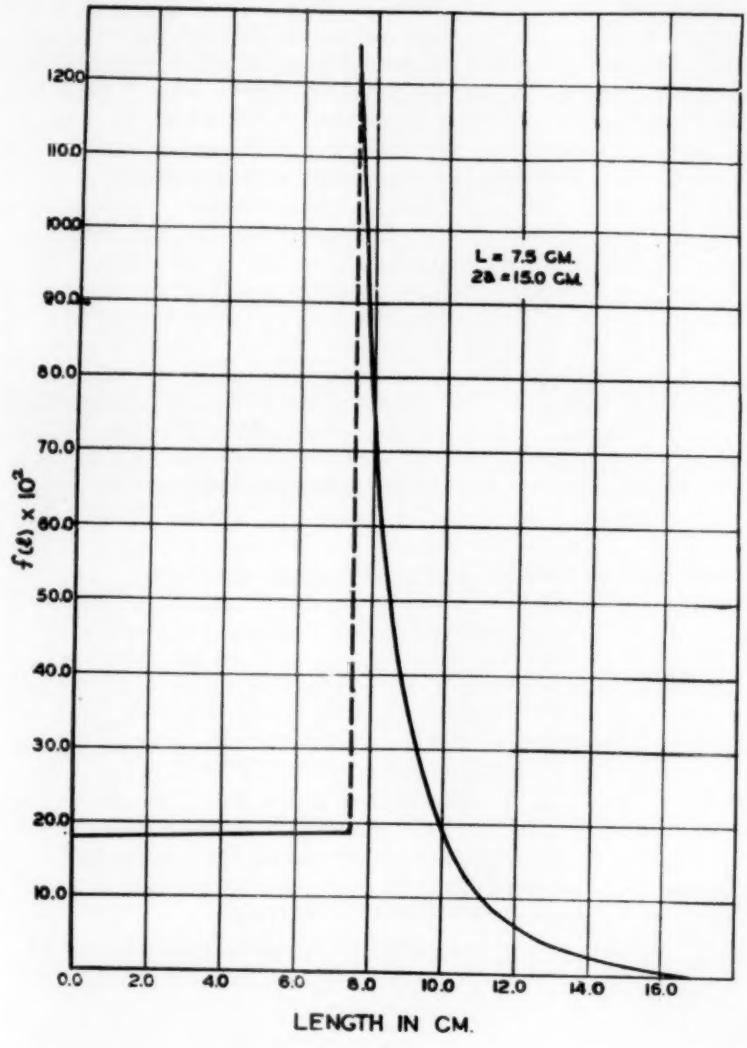
FIG. 2*A*.




path produced by secondary cosmic rays. The purpose of the present paper is simply to deduce the formulæ.

In conclusion, I wish to express my appreciation of the services of my research assistant Mr. W. E. Ramsey, who has checked all of the mathematical formulæ and computations.

FIG. 2B.


L = 7.5 CM.
2a = 15.0 CM.
f(ℓ) x 10²
120.0
110.0
100.0
90.0
80.0
70.0
60.0
50.0
40.0
30.0
20.0
10.0
0.0
2.0
4.0
6.0
8.0
10.0
12.0
14.0
16.0
LENGTH IN CM.

**A Novel Fire Detector.**—At the coming British Shipping Engineering and Machinery Exhibition there will be displayed a "detector" which operates almost instantaneously on the outbreak of fire. The device consists of two identical phosphor-bronze bridges so constructed as to have an equal curvature. One bridge, however, is protected by a heat-insulating sleeve, while the other has no thermal protection. Any sudden rise of temperature expands the unprotected bridge more than that which is thermally protected and in consequence an electrical contact is made. A slow rise of temperature expands the two bridges uniformly and thus maintains their relative separation. Thus the instrument remains equally sensitive under widely varying conditions of normal temperature. Once it is installed, a setting suitable in the Arctic Circle remains equally so in the Equatorial region. An additional contact may be introduced to take care of any undue slow rise of temperature up to 200° F. As an indication of the extreme sensitivity of this instrument, it is only necessary to breathe upon it to close the circuit. The material of construction renders the device impervious to rough handling or corrosive atmospheres.

C.

**Another Method for Producing Artificial Daylight.**—The Official Bulletin of the British Shipping Exhibition announces an exhibit of the latest form of artificial electric lighting which offers the advantages of artificial daylight coupled with 40 per cent. saving in current consumption and a light non-injurious to the eyes. By means of specially designed colored mirror reflectors, the light emitted corresponds in spectrum and effect to that of daylight. Unlike the bluish, so-called daylight emitted by the majority of daylight lamps, these reflected rays give a pale but cheerful sunlight under which all objects appear in their true colors. Owing to the scientifically designed mirror reflectors incorporated in all lighting units, it is possible to use 60-watt bulbs in place of 100-watt and still obtain the same intensity of lighting.

C.

# THE DISTRIBUTION OF COSMIC RAY PATHS IN A VERTICAL CYLINDER.

BY

DAVID L. MACADAM and M. REUEL LIPMAN.

BARTOL RESEARCH FOUNDATION

Communication No. 79.

The primary purpose of the present paper is to construct for a vertical cylinder of length $L$ and radius $a$, a curve showing the relation between $f(l)$ and $l$, where $f(l)dl$ is the number of cosmic rays passing through the cylinder and having therein lengths between $l$ and $l + dl$. In other words, the ultimate purpose of the paper is the same as that of the paper by Dr. W. F. G. Swann published in this JOURNAL under the same title.[1] The scheme of procedure was suggested by Dr. Swann in graphical form, partly to provide a check upon the analytical calculation, and partly because it represents a method of procedure applicable when the integrals of the analytical method are not obtainable.

It is further possible to develop the graphical method to a partly analytical form in which the plan of calculation follows a plan somewhat different from that given in Dr. Swann's treatment, but a plan leading, of course, to the same ultimate results. The transformation of the graphical method to this analytical form has been carried out by one of us,[2] for a portion of the curve, and the results and procedure will be sketched in the last paragraph.

## GRAPHICAL SOLUTION.

The assumption as to the distribution of cosmic rays in space, and the notation adopted, is the same as that used in Dr. Swann's paper.[1] In other words, if $\theta$ is the angle from the vertical, and $\varphi$ is the azimuthal angle, it is assumed that the number of rays $\delta n$ falling within the solid angle $\sin\theta\, d\theta\, d\varphi$ upon element $ds$, perpendicular to the axis of the solid

[1] *Journal of The Franklin Institute*, this issue, page 559.

[2] D. L. MacAdam.

angle, is

$$\delta n = A \sin\theta \cos^2\theta \, d\theta \, d\varphi \, ds.$$

The general procedure was then as follows:

1. Find the projected area $S$ of that part of the cylinder within which the rays lying between $\theta$ and $\theta + d\theta$ from the vertical, and incident on $S$, have path lengths greater than, or equal to, $l$. The ray direction is taken as the direction of projection.

2. Find the number of rays per second, $\delta N_{\theta,\, l}$, from directions between $\theta$ and $\theta + d\theta$ from the vertical, having path lengths inside the cylinder greater than, or equal to, $l$. This number is $2\pi AS \sin\theta \cos^2\theta \, d\theta$.

3. Find the total number of rays per second, $N_l$ which have path lengths inside the cylinder greater than, or equal to, $l$. This is found by integrating the result of step (2) with respect to $\theta$ between the limits fixed by the geometry of the figure, and beyond which the area expressions are not valid.

4. Find the number of rays per second $f(l)dl$ which have path lengths in the cylinder between $l$ and $l + dl$. This is found by differentiating the integrals of step (3) with respect to $l$.

*Expression for the Area S.* Referring to the shaded portions of Fig. 1, which represent the portions of the cylinder in which the path length is greater than, or equal to, $l$, it is easy to see that the desired projected area is the stippled area $S$ in the auxiliary view. This area is $UV$ + (projected area of $WXYZ$).

$$U = 2a\cos\psi \qquad V = (L - 2l\cos\theta)\sin\theta$$

$$UV = 2aL\cos\psi\sin\theta - 4al\sin\theta\cos\theta\cos\psi.$$

$$\text{Area } WXYZ = (a^2/2)(2\pi - 4\psi) + 4\left(\frac{a^2\sin\psi\cos\psi}{2}\right)$$

$$= a^2(\pi - 2\psi + \sin 2\psi).$$

Projected Area $WXYZ = a^2\cos\theta(\pi - 2\psi + \sin 2\psi)$. Total Area of Projection $= S = 2aL\sin\theta\cos\psi + a^2\cos\theta(\pi - 2\psi - 3\sin 2\psi)$.

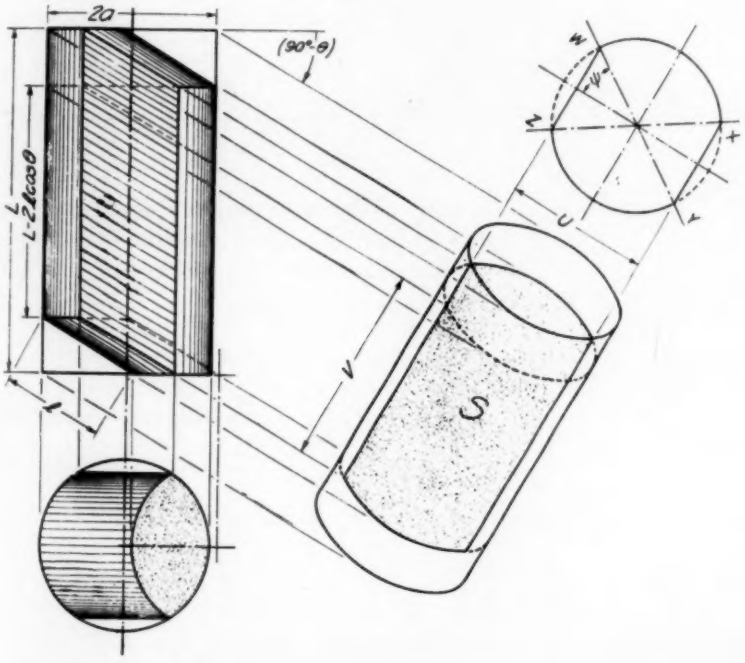
The limits of validity of this formula are fixed by the

geometry of the figure and are easily seen to be:

$$\mu = \text{upper limit of } \theta = \begin{cases} \pi/2 \text{ when } K \leqq 1 \\ \sin^{-1} 1/K \text{ when } K > 1 \end{cases}$$

$$\lambda = \text{lower limit of } \theta = \begin{cases} 0 \text{ when } M/K \geqq 1 \\ \cos^{-1} M/K \text{ when } M/K < 1. \end{cases}$$

FIG. 1.




In this and all following expressions, the abbreviations $K = l/2a$, and $M = L/2a$ have been used. Beyond the assigned limits, the area of projection is zero.

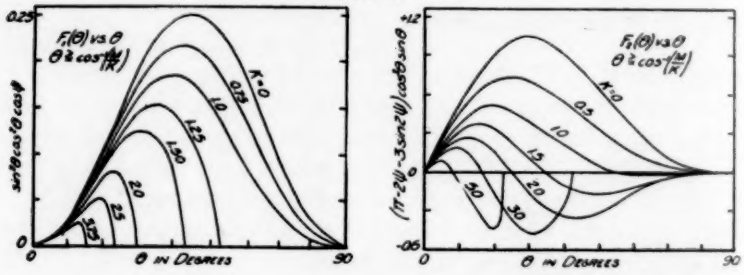
The value of $\delta N_{\theta, l}$ referred to under (2) is

$$\delta N_{\theta, l} = 2\pi A[\{2aL \sin\theta \cos\psi\} + \{a^2(\pi - 2\psi - 3 \sin 2\psi) \cos\theta\}] \cos^2\theta \sin\theta \, d\theta.$$

Writing

$$F_1(\theta) = \sin^2\theta\cos^2\theta\cos\psi;$$
$$F_2(\theta) = (\pi - 2\psi - 3\sin 2\psi)\cos^3\theta\sin\theta;$$
$$\delta N_{\theta,\,l} = 2\pi A[2aLF_1(\theta) + a^2F_2(\theta)]d\theta.$$

FIG. 2.




a. b.

The quantities $F_1(\theta)$ and $F_2(\theta)$ are plotted separately in Figs. 2*a* and 2*b* in order to facilitate the subsequent development to cylinders of various sizes with the minimum of labor. The value of $N_l$ referred to in (3) is given by

$$N_l = \int_\lambda^\mu 2\pi A\{2aL\cos^2\theta\sin^2\theta\cos\psi + a^2(\pi - 2\psi - 3\sin 2\psi)\cos^3\theta\sin\theta\}d\theta,$$

where

$$\lambda = \cos^{-1} L/l \qquad \text{if } L/l < 1$$
$$\lambda = 0 \qquad \text{if } L/l \geqq 1$$
$$\mu = \sin^{-1} 2a/l \qquad \text{if } 2a/l < 1$$
$$\mu = \pi/2 \qquad \text{if } 2a/l \geqq 1.$$

Substituting $\sin\psi = K\sin\theta$ and $\cos\psi = \sqrt{(1 - K^2\sin^2\theta)}$, and $\sin 2\psi = 2\sin\psi\cos\psi$, where $K = l/2a$, we have $N_l = 2\pi A[2aLA_0 + a^2B_0]$, where

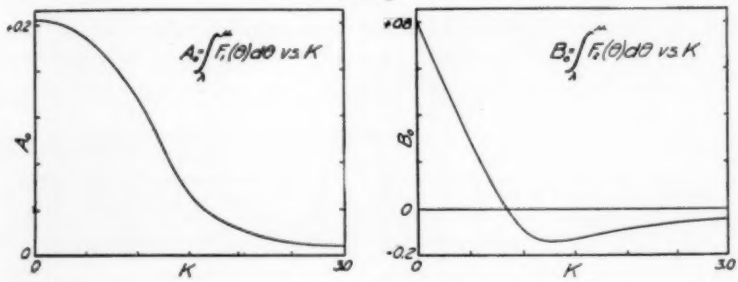
$$A_0 = \int_\lambda^\mu \sqrt{(1 - K^2\sin^2\theta)}\cos^2\theta\sin^2\theta\,d\theta$$

$$B_0 = \int_\lambda^\mu [\pi - 2\sin^{-1}(K\sin\theta) - 6K\sqrt{(1 - K^2\sin^2\theta)}\sin\theta]\cos^3\theta\sin\theta\,d\theta.$$

$A_0$ and $B_0$ may be obtained by graphically integrating the curves given in Figs. 2*a* and 2*b* for different values of $K$. We must distinguish two cases.

*Case 1, where* $l < L$. Here the integration is carried out from $\theta = \lambda = 0$ to $\theta = \mu$, where the limits $\mu$ are automatically designated as the points at which the curves finally cut the $\theta$ axis, since it is easy to see that $F_1(\theta)$ and $F_2(\theta)$ are zero at the values of $\mu$ given above as the limits. In this way, for

FIG. 3.




*a.* *b.*

$l < L$, the curves in Figs. 3*a* and 3*b* are obtained and these are applicable for all cylinders within the range of dimensions specified by the maximum value of the abscissa. Again, for $l < L$, the number of paths traced per second with lengths between $l$ and $l + dl$ is $f(l)dl$, where [3]

$$f(l)dl = -\frac{dNl}{dl}dl = -\pi Aa\left[\frac{2L}{a}\frac{dA_0}{dK} + \frac{dB_0}{dK}\right]dl. \qquad (1)$$

From curves 3*a* and 3*b* we can construct $dA_0/dK$ and $dB_0/dK$, and plot them against $K$. This is done in Figs. 4*a* and 4*b*. The resulting curves then provide for any of the cylinders the values which, when substituted in (1), give $f(l)$ for $l < L$.

*Case 2, where* $l > L$. For this case, Figs. 2*a* and 2*b* still apply as representatives of $F_1(\theta)$ and $F_2(\theta)$; but the lower limit of integration, instead of being zero, is now a quantity characteristic of the particular cylinder concerned, and given,

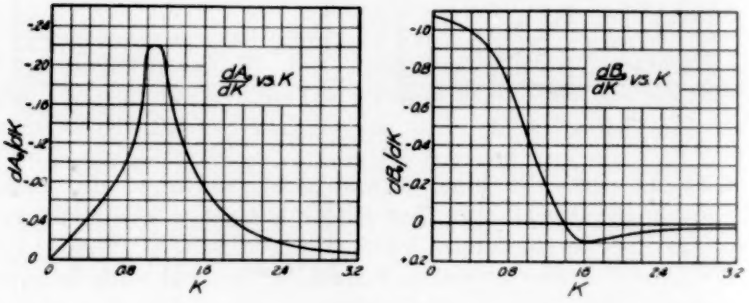
[3] The minus sign is incorporated to take account of the fact that increasing $l$ diminishes $Nl$.

as above, by $\lambda = \cos^{-1} L/l$. We must, therefore, corresponding to Figs. 3*a* and 3*b*, and to Figs. 4*a* and 4*b*, construct curves which are special for the cylinder concerned. With these curves constructed, the procedure is as outlined under Case 1.

*The Discontinuity in $f(l)$ at $l = L$.* The foregoing procedure will reveal an actual discontinuity in $f(l)$ at $l = L$, a discontinuity whose origin may be seen, and whose magnitude may be accurately calculated as follows.

Let us put $x = \cos\theta$ in $A_0$ and $B_0$ so that the lower limit of $x$ in each is now $x_0 = L/l$. The expressions $dA_0/dK$ and $dB_0/dK$ now each involve two parts, a part resulting from differentiating the integrand and a part resulting from differ-

FIG. 4.

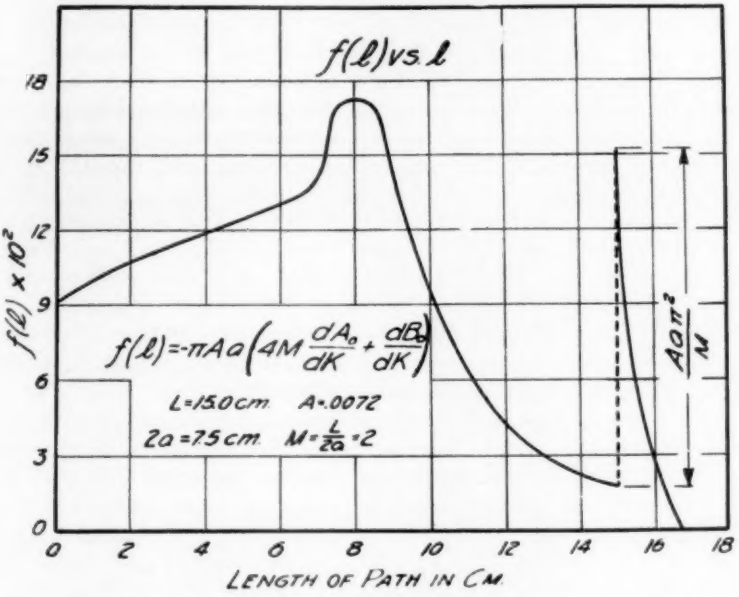



entiating the limits. The latter part gives rise to the discontinuity of $dB_0/dK$ because it is zero for $L/l > 1$, and, for $L/l$ just less than unity, it is equal to minus $(dx_0/dK)_{x_0=1}$ times the value obtained by substituting $x_0 = 1$ in the said integrand. Since $dx_0/dK = -2aL/l^2$, the discontinuity in $dB_0/dK$ is consequently $-2aL\pi/l^2 = -2\pi a/L$ at $l = L$, so that the discontinuity in $f(l)$ is $2\pi^2a^2A/L$ as far as the contribution by $B_0$ is concerned. Since the integrand in $A_0$ is zero at the lower limit, it contributes no discontinuity.

In accordance with the foregoing principles, the curve relating $f(l)$ to $l$ has been constructed in Fig. 5 for the case of a cylinder in which $L = 4a$. This is the case given in the paper by Dr. Swann referred to above, and the curve, of course, fits his curve.

*Analytical Solution.* It is, of course, possible to carry out the foregoing operations analytically without appeal to graphical methods, although this might not always be possible or convenient with the choice of other forms of the cosmic ray distribution law in space. The procedure involves forming the analytical expressions for $dA_0/dK$ and $dB_0/dK$ which take the form of integrals in $\theta$, involving elliptic integrals in the case of $dA_0/dK$ and elementary functions in the case of $dB_0/dK$. The details of the analysis are more intricate for the

FIG. 5.




case $l > L$ than for $l < L$. For the latter case, they have been carried out by one of us, with the resulting realization of a formula for $f(l)$ which is the same as that given by Dr. Swann in the paper already referred to. While the analytical solution is more exact for computational purposes, the curves in Figs. 4*a* and 4*b*, when used in conjunction with equation (1), form a very rapid method of determining $f(l)$ for the case $l < L$.

We wish to thank Dr. Swann for his interest and helpful suggestions during the preparation of this paper.

**An Ill Wind That . . . .**—(U. S. Dept. of Agric. Clip Sheet No. 789.) The direction of the wind is of extreme importance to airplane pilots. A tail-wind represents a saving in time and fuel and aviators continually rely upon the Weather Bureau for information regarding satisfactory air currents. An illustration of to what good use such information can be put will be found in a fairly recent case where a northbound pilot flying the Pacific coast route with a 40-mile tail-wind passed 4,000 feet below a southbound pilot also getting a 40-mile tail-wind.

C.

**Mercury in Human Metabolism.**—PAUL BORINSKI (*Klinische Wochenschrift*, 1931, X, 149–153) has made an elaborate study of the mercury content of the ingested food and of the excretions of man. The mercury content of 20 foods was determined; none was found in apples, tomatoes, and pig liver; the amount found in the other foods was as little as 0.00016 mg. of mercury per kilogram of carrots or of pork, and as great as 0.00265 mg. and 0.0047 mg. of mercury per kilogram of bread and of fresh herring, respectively. The mercury apparently had its origin in the soil, for 5 samples of soil were found to contain between 0.00033 and 0.0032 mg. of mercury per kilogram of soil. From the analyses of the foods, the mercury content was calculated for each of two diets, which were intended for an adult and supplied an adequate quantity of protein as well as sufficient calories. Each diet supplied approximately 0.005 mg. of mercury to the human body daily.

The mercury content of the excretions of over 400 persons was also determined. When a person was not in contact with mercury as an industrial hazard, and either had no amalgam fillings in his teeth or else such fillings had been inserted for at least 3 months, then the daily excretion of mercury was of the order of magnitude of 0.005 to 0.010 mg. daily. The daily intake and the daily excretion of mercury were then approximately equal. Persons in contact with mercury, and those with new (less than 3 months old) amalgam fillings showed a higher output of mercury. Amalgams of either copper or the noble metals, when inserted as fillings, ceased to serve as a source of mercury at the end of 3 months. The mercury in the food is viewed as a normal constituent which is not injurious to health; its physiological function is uncertain. Mercury was found in the human liver, at birth and in the adult.

J. S. H.

# THE PHOTOGRAPHIC PROPERTIES OF SILVER SULFIDE.*

## Part 3. The Print-out Effect—Photographic Aspect.

BY

K. HICKMAN AND W. WEYERTS.

We have recorded elsewhere [1] that finely divided silver sulfide will darken further when suspended in certain solutions of silver salts and exposed to light. During this treatment the silver sulfide appears to suffer comparatively small change and its darkening is caused by the deposition of metallic silver in the immediate neighborhood of the particles. This silver has greater optical covering power, mass for mass, than the sulfide, and, since it can accumulate to 5–10 times the quantity of sulfide, the increase in optical density is great. The effect is retarded by gelatin, but is never entirely repressed. It is exhibited by the sulfided image of ordinary photographic plates and papers and the treatment of these images offers a means of intensification.

Since the reduced silver is being supplied from the solution the process should be classed as physical intensification, but it differs from the usual physical intensification in an important particular. Physical intensifiers have a "tendency to deposit" which they exert uniformly over the whole photographic surface, and the operator trusts that the tendency will be manifested only where there is a seeding agent—properly, the silver image. Unfortunately, many agencies start the deposition of silver and these are often present in the clear portions of negatives and prints, so that it is common experience that physical intensifiers stain badly. The solutions in the process under discussion have no spontaneous tendency to deposit and the tendency is generated only where the light is caught by the image; elsewhere, the solution remains inactive and unstaining. Very great differentiation exists between image and clear portions, and, consequently, high gamma increases can be secured without fog or stain.

* Communication No. 522 from the Kodak Research Laboratories.

In searching for a name for this kind of image building the obvious "sulfide intensification" is rejected because the phenomenon is a general one, chloride, bromide, iodide, and thiocyanate images all building up to a certain extent under light action in appropriate solutions. We suggest "optical intensification" as less objectionable (if less accurate) than names incorporating luminous, radiant or photo; and it is by this provisional name that we shall refer to the process.

### A CAUTION.

The present intensification method appears so versatile that it may be thought that a universal process has been disclosed, applicable to most photographic materials and problems.

In its present state of development this is emphatically not the case and much work remains to be done before reliable recipes can be offered to the practical worker. Different materials behave differently when treated by the process, and for the present we prefer to offer only an account of the phenomena observed, reserving exact working directions for a separate communication.[7]

### THE GENERAL CASE.

The principle underlying optical intensification (or the reduction of a soluble silver salt in light through the agency of an insoluble salt) has long been used in photography. Silver print-out papers are compounded, broadly speaking, of two parts, one an insoluble light-sensitive silver salt, and the other a semi-soluble insensitive* silver salt matrix. The sensitive salt AgX (often a silver halide) reacts with the insensitive salt AgY (perhaps a citrate or oxalate) thus:

$$AgX + \text{light} \rightarrow Ag_{\text{metal}} + X$$

and

$$X + AgY \rightarrow AgX + Y,$$

re-forming the AgX for further use in receiving light. The acid radicle Y cannot, of course, exist alone, and is chosen

* Insensitive only to the wave-length for which the paper is used. Even soluble silver salts are sensitive to the appropriate ultra-violet radiation.

from a class which will decompose spontaneously or else be absorbed by an acceptor added for the purpose.

If, then, the salt AgX is a small, but essential, part of the print-out emulsion, surely a faint photographic image composed of AgX, immersed in a solution which contains the rest of the mixture, should print out in light with a vigor corresponding at each point roughly with the density of the original AgX—the AgX image should be optically intensified? We shall show that this happens readily with silver sulfide, and also that images of silver chloride, bromide, iodide, azide, and thiocyanate, to mention the common insoluble silver salts, also intensify.

Now, in each case the primary silver salt image must be formed from metallic silver in the gelatin layer by the action of soluble salts, and there will be much residual salt to wash out and there will remain a small quantity of chloride from the tap water which is not removable except by very special treatment.[2] When the converted layer is placed in the silver solution, a little chloride or other silver salt is formed which, owing to its finely divided condition, is relatively more sensitive than the image proper, and the material may stain under illumination.

The insoluble silver salts mentioned above are stable compounds with considerable heats of formation and they are substantially white in color. Corresponding with these characters, the long wave-length limits of their print-out sensitivities are in the blue. The approximate limits for the silver halides are well established,[3] and only a partial extension towards the red end [4] of the spectrum is conferred by the silver produced in the early stages of exposure.

Silver sulfide, on the other hand, has a small heat of formation, is dark colored, and is correspondingly panchromatic, the sensitivity extending from the ultra-violet to 1.3 $\mu$. It is relatively sensitive to the tungsten lamp and it permits optical intensification by rays which have little effect on residual chloride.

The disposal of residual sulfide before exposure is also simpler. Whereas the halogens have to be removed before they are out of action, sulfide ions are readily converted to harmless sulfate or thionates.

The infra-red sensitivity and the freedom from chemical stain puts the sulfide in a class by itself as a base for intensification.

### THE PROCESS IN OUTLINE.

Optical intensification is applicable to ordinary negatives and lantern slides, to bromide prints and other prints made on developing-out papers. The photograph is bleached in the usual bromide-ferricyanide mixture, or preferably in a permanganate-bromide solution. It is then washed and darkened in dilute sodium sulfide or in a sulfide solution containing ammonia, hypo or other silver halide solvent. It is rinsed, immersed in dilute sodium bisulfite for a minute to destroy the residue of soluble sulfide, and is then ready for intensification. The intensifier can consist of sodium-silver-nitrite solution or of sodium silver sulfite. The negative or print is immersed in the solution and rocked gently in sunlight or close under a tungsten lamp for 2–10 minutes according to the density increase required. It is then transferred to a plain sulfite solution containing hypo and is washed for the usual negative or print-washing time.

### THE PROCESS IN DETAIL.

The authors can find no arrangement of the mass of experimental material, rapidly accumulating, which presents the subject in an entirely satisfying sequence. Among the many factors determining the degree of intensification are the following:

1. The kind of emulsion.
   - (*a*) Grain size distribution,
   - (*b*) nature and condition of the gelatin,
   - (*c*) the relative proportions of silver halides and gelatin in the emulsion.
2. The developer for the silver image.
3. The bleaching bath.
4. *The sulfiding bath.*
5. The intermediate washing.
6. *The intensifying bath.*
7. The color and intensity of the exposing light.
8. The temperature.

On some of these points the following comments are offered:

1*a*. Fine-grained emulsions intensify best, the unwashed variety used for certain warm toned developing-out papers being most amenable.

1*c*. With both nitrite and sulfite baths, gelatin is an inhibitor so that the less there is present the faster the intensification. The sulfite reaction may depend on the accumulation of an unstable intermediate product which reaches the critical concentration sooner when the silver sulfide is housed in a relatively small jelly space.

2. A trustworthy comparison of developers would require a separate communication and is outside the scope of this paper. Developers, such as p-phenylene diamine, which disperse the grains, favor intensification. Little difference could be found between the normal M.Q. formulæ and the fine-grained borax developers.

3. The reaction is repressed by many metallic impurities. Ferricyanide, unless removed by an hour's washing, yields an inferior image. Dilute permanganate-bromide baths have been found suitable.

4. The bleached silver halide grains can be darkened to an almost colloidally dispersed sulfide by incorporating a solvent (hypo, ammonia, or thiocyanates) in the sulfiding solution. It would be anticipated that the great extension of reactive surface would hasten intensification. In actual fact, the dispersed images are faster in silver sulfite solutions and slower in nitrite; and the complete chemistry of the difference has not yet been explored. Since the sulfiding has more effect on the speed of action and the shape of the characteristic curve than all the other factors added together, it is necessary to interpolate some later findings here in the form of Table I, which records the properties of *three* well defined *classes* of sulfided images. *It is by these classes that the images will be referred to subsequently.*

The truest comparison between nitrite and sulfite intensifiers would be made with the ordinary sulfide image. Since, however, the most useful process employs the diffused type 3 sulfide, much of the work detailed hereafter refers to sulfite

TABLE I.

*Typical Sulfiding Baths.*

| Reference Number. | Composition. | Condition of the Sulfided Image. | Action of Nitrite Intensifier. | Action of Sulfite Intensifier. |
|---|---|---|---|---|
| 1. | Sodium sulfide 2 gms. per liter | Original grain form intact. Cold brown tone | Good intensification to a $\gamma$ increase of about 3. Black deposit. All densities intensified | Good intensification to a $\gamma$ increase of about 3. Brown-black deposit. All densities intensified. Speed of action greatly inferior to best nitrite speed |
| 2. | Sodium sulfide 2 gms. per liter; hypo, 10 gms. per liter | Slight colloidal dispersion round grains. Resolving power of material scarcely affected. Warm brown tone | Intensification slowed down. All densities increased. Black deposit | Intensification greatly hastened to a $\gamma$ increase of 5 or 6. Fog densities not intensified. Warm brown tone |
| 3. | Sodium sulfide, 2 gms. per liter; hypo, 10 gms. per liter; ammonia (conc.), 10 cc. | Highly dispersed yellow image; resolving power reduced to about half | Poor intensification of all densities | Extreme intensification to a red brown image. Fog densities not affected |

intensification only. A more trustworthy comparison of the two intensifying solutions is to be found in the study of precipitated $Ag_2S$ described in the earlier paper.

The other subjects mentioned on p. 5 will be dealt with as they arise.

## EXPERIMENTAL.

The photographic material used, unless otherwise mentioned, was sheet nitrate film coated on one side only with half the usual thickness of motion picture positive emulsion. The wedges for intensification were made from contact exposure with one of two master wedges, the first a small tablet yielding an image, after 1½ min. development in D–16 (half-strength),* of the shape shown in Fig. 1*a*, the second giving the longer wedge of 9 steps, Fig. 1*b*.

These were bleached in

| | |
|---|---|
| Water, to | 1 liter |

then added in order,

| | |
|---|---|
| Potassium permanganate | 3 gms. |
| Sodium acetate | 10 gms. |
| Potassium bromide | 5 gms. |
| Acetic acid | 5 cc. |

rinsed, cleared in

| | |
|---|---|
| Water | 1 liter |
| Sodium bisulfite | 50 gms. |

and sulfided in one of the various solutions.

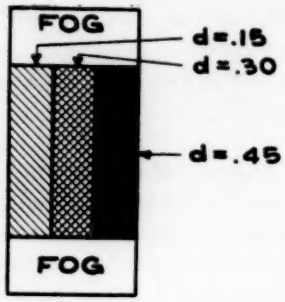
The strips were exposed in test tubes full of intensifier which were immersed in a glass cell (Fig. 1-*C*) through which water at constant temperature was circulated. The lamp, a 115-volt 250-watt tungsten projection type working at 110 volts, was held at 6 inches to 1 foot distance.

---

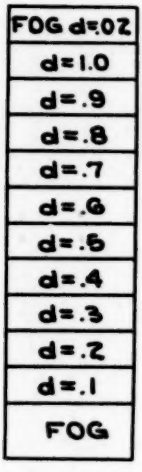
* *Full Strength D-16 Developer.*

| | |
|---|---|
| Elon | .31 g. |
| Sodium sulfite | 39.6 |
| Hydroquinone | 6.0 |
| Sodium carbonate | 18.7 |
| Potassium bromide | .86 |
| Citric acid | .68 |
| Potassium metabisulfite | 1.5 |
| Water, to | 1 liter |

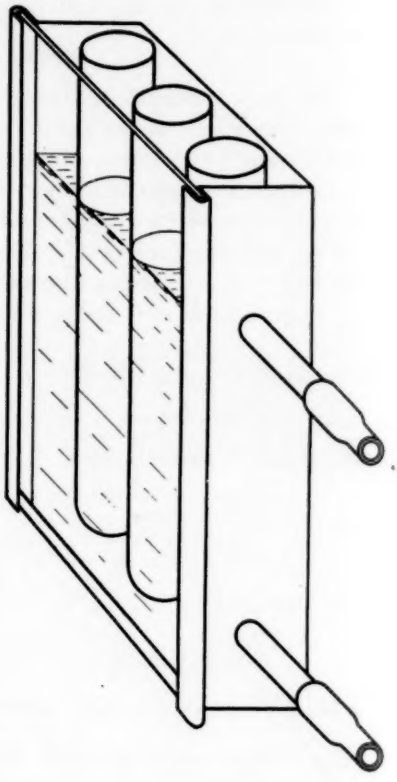
FIG. 1.




(Fig. 1*a*)




(Fig. 1*b*)

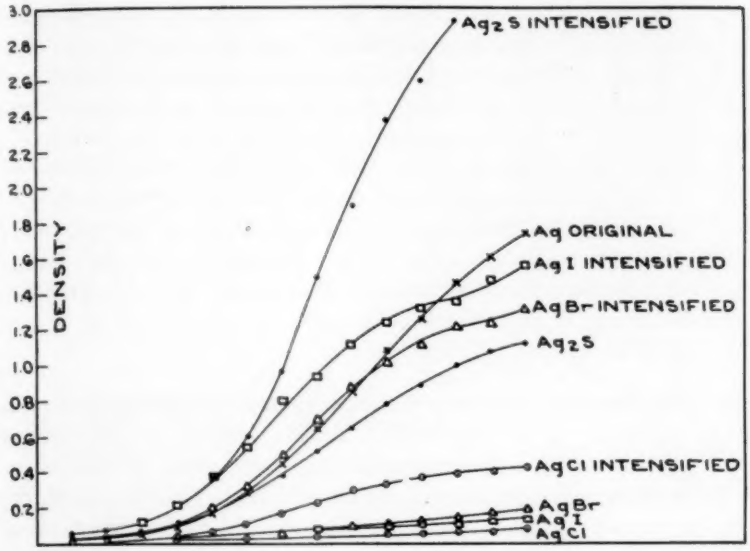


(Fig. 1*c*)

**DENSITOMETRY.**

The most elementary question to be answered is whether optical intensification preserves tone reproduction, leaving the shape of the characteristic curve intact whilst it is altering the slope. The answer, speaking broadly, is that the sulfide images yield faithful density increase, improving, if anything, the shape of the characteristic curve, but the halide images give distorted rendering. This is shown in Fig. 2 where the diffuse densities of original silver image wedges are compared with

FIG. 2.




similar images, converted to chloride, bromide, iodide, and sulfide (type 2)* and intensified in

| | |
|---|---|
| Sodium sulfite | 40 gms. |
| Silver nitrate | 8 gms. |
| Sodium bisulfite | 15 gms. |
| Water | 1 liter |

by exposure for 3 minutes, about 18″ below a Sun Lamp (G. E. type S.1 tungsten filament + mercury vapor). This lamp was chosen for this experiment only in order to give the halogenized strips sufficient ultra-violet illumination to compete with the red sensitiveness of the sulfide. The superiority of the latter both in degree and truthfulness of increase is apparent and would have been more marked had the exposure been to the 250-watt ordinary tungsten lamp.

Where the shape of the characteristic curve is roughly preserved the intensification can be expressed as an increase of gamma and the increase can be deduced from the slope † of

* Azide, thiocyanate, and other insoluble images were also found to intensify.

† It is important that the slope of the curve be taken from the region where the intensification is to be utilized. Thus, if the toe densities are being intensified, it is the slope of the toe and not the normal straight line portion that is significant.

the curves before and after treatment, or from the ratio of the two densities of a chosen image step. Thus, an image having a gamma of .7 before treatment and 1.4 after would have received an intensification of 2. In this notation the absence of intensification is unity, rather than zero, and reduction effects are expressed in fractions of the original gamma or density.

At the toe of the characteristic curve, however, the densities seldom intensify uniformly, those below .15 often failing to respond at all, and those between .2 and .4 increasing relatively more than their denser neighbors. In these cases the change of slope becomes meaningless and the degree of intensification (or reduction) is expressed as a multiple (or fraction) of the original density in question.

### THE INFLUENCE OF SOLUTION CONCENTRATION ON THE RATE OF INTENSIFICATION.

Two important processes are at work during optical intensification. There is the influx of fresh solution and the effusion of old; and there is the absorption and utilization of light by the sulfide image. To these may be added a third process, sometimes in evidence, which is the accumulation of a reducing substance preliminary to the deposition of silver.

Two limiting cases are possible, one where the supply of radiant energy is slow compared with diffusion and the progress of intensification is determined solely by the light available to the various densities; the other, where the light is in great excess and the increase in density is dependent on the counter diffusion of new and old solution.

Manifestly, the lightest densities are most favorably situated since all their particles receive all the available light and they make little demand on the solution. The heavy densities would be expected to intensify comparatively little since their lower components are in the shade of the upper. It is found, experimentally, that the silver halide images show the expected falling off in the upper densities but the sulfide images increase unduly.

The passage between the two limits: sufficient reactants—insufficient light and sufficient light—insufficient reactants, can be accomplished by choosing a suitable constant value of illumination and varying the concentration of solution. A

suitable intensity was secured at 6″ from the 250-watt lamp with strips exposed at 60° F. in the water cell; and the solution was varied from the most concentrated mixable to such dilutions that intensification practically ceased.

Three different tests were made:

(*a*) with sulfite solution on a plain sulfide image (type I sulfiding, of Table I), see curves, Fig. 3.
(*b*) with sulfite solution on type 3 sulfide image; see curves, Fig. 4.
(*c*) with nitrite solution on type 1 image; see curves, Fig. 5.

The most concentrated solution in each case contained 20 grams of silver nitrate per liter, and thus required approximately 100 grams of neutral sulfite-bisulfite mixture to give a stable solution; or 200 grams of sodium nitrite. The solutions were diluted, step by step, to form an exponential series in which the fourth step was a solution 1/10th the original concentration and the eighth step, one of 1/100th. Both concentrations and normalities (with respect to Ag) are shown in the graphs.

The type 1 images respond similarly in either solution, although in nitrite the darkening is about 10 times more rapid. The lesser densities intensify most and continue intensification in solutions too weak to affect the upper densities appreciably; and such dilute solutions can be used to lift the toe of a poor characteristic curve without substantially increasing the average gamma. The most interesting point is that moderately dilute solutions are the most active, all the curves showing a maximum at approximately 5 grams of silver nitrate per liter. There is no evidence that this is due to hindered diffusion caused by the repression of the swelling of the gelatin in the concentrated solution. Such an assumption would demand that the diffusion was cut 20 times in the most concentrated solution, since the speed is about the same as in the solution 20 times more dilute.

The half per cent. solutions are therefore intrinsically faster. It was suggested in a former paper [1] that the speed of darkening is proportional to the silver ion concentration. The silver ion concentration is probably fairly constant (no meas-

urements have yet been made) over this part of the dilution range, but the competition between silver ions and other ions and molecules at the active silver sulfide surface is probably so intense in the most concentrated solution that the absorption of silver ion is diminished.

In the very dilute solutions the quantity of silver deposited

FIG. 3.

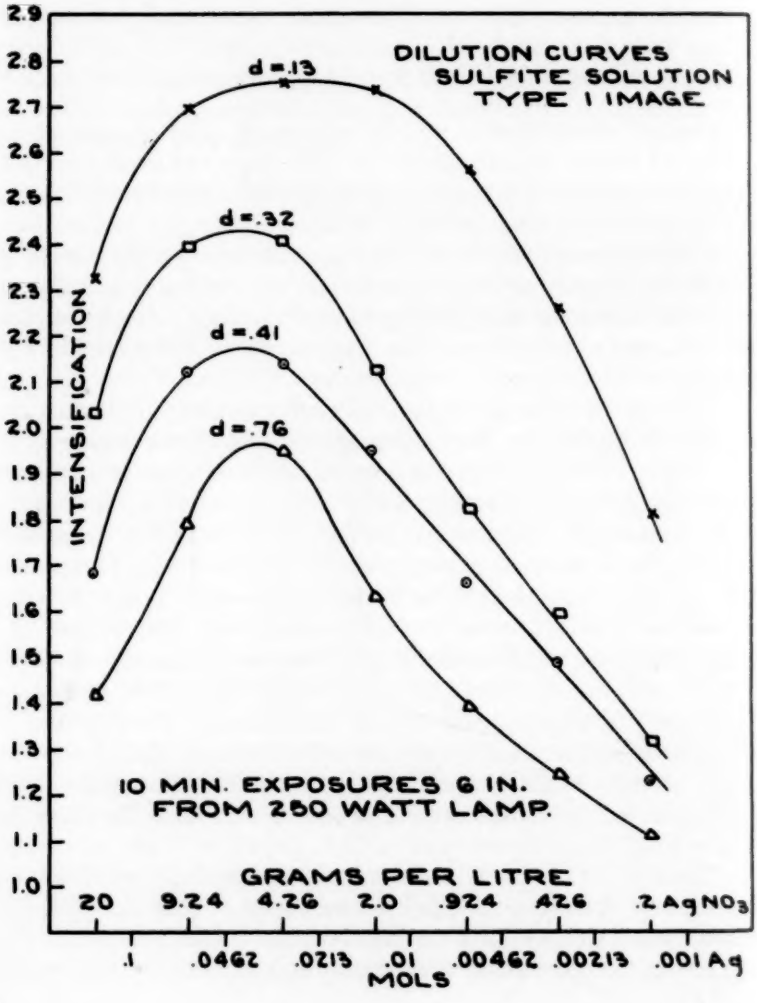
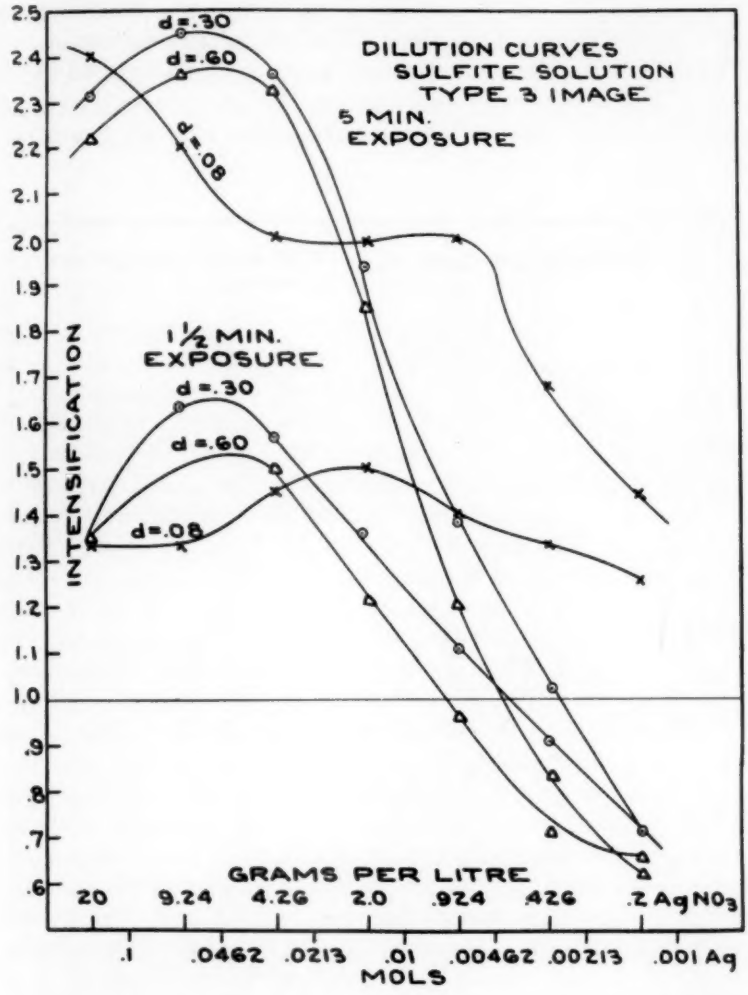


FIG. 4.




is independent of the density of the sulfide image and is determined by the rate of supply of soluble silver.

The *net* density gain for three sets of densities is given in Table II, which is compiled from the same data as the curves in Figs. 3 and 5. Sulfite solutions containing less than .5 gm. $AgNo_3$ per liter raise all the densities from .1 upwards by an

equal increment and thus give *no* increase of gamma. The same is true of nitrite solutions containing less than 1 gram $AgNO_3$ per liter. This limit of selective intensification is, evidently, variable with the light intensity and time of exposure. The weaker the illumination the less concentrated the solution would need to be to differentiate between the densi-

FIG. 5.

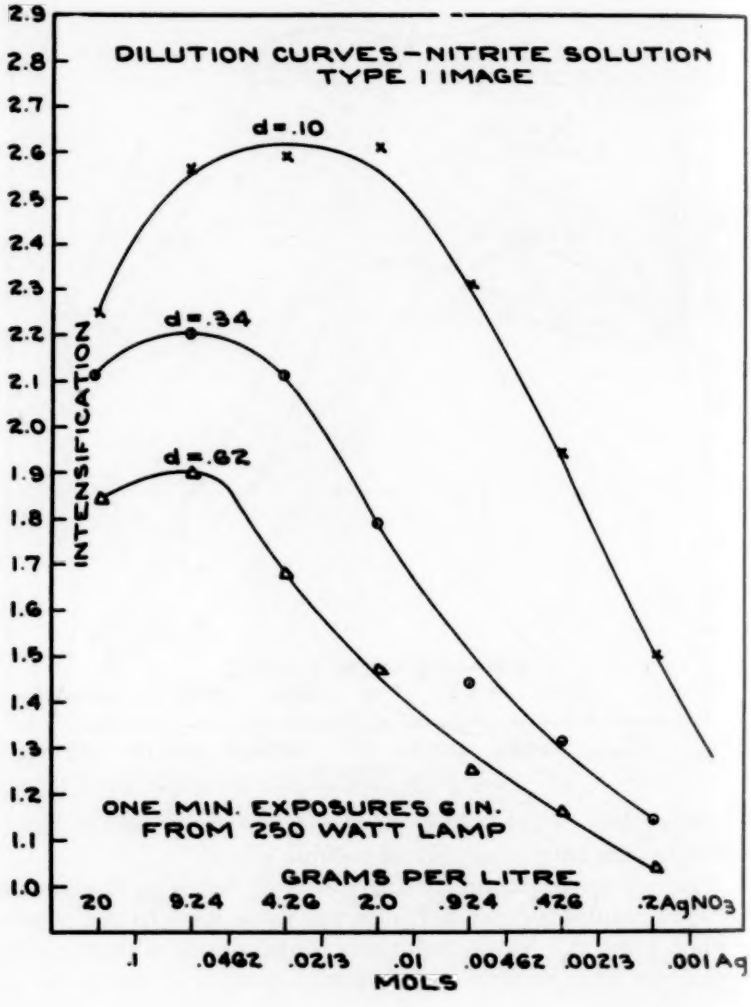


TABLE II.

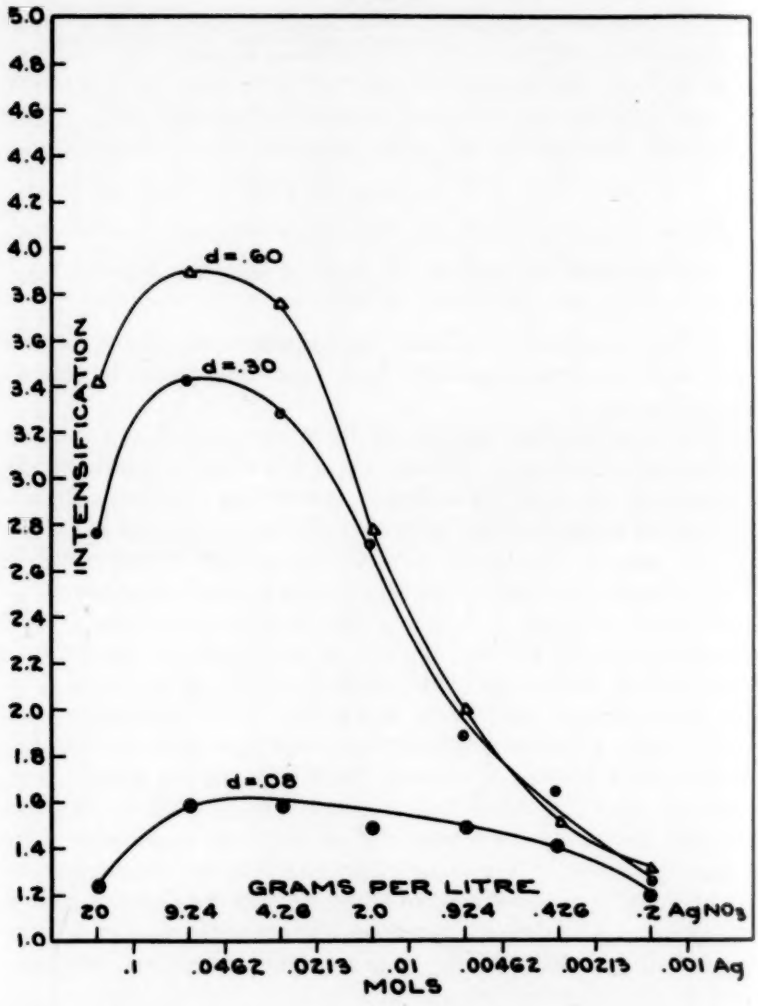
| Type of Intensifying Solution. | Average Original Sulfide Densities. | Concentration of Silver Nitrate in Grams per Liter; Density Increases. | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | 20. | 9.2. | 4.3. | 2.0. | .92. | .43. | .2. |
| Sulfite | .16 | .20 | .21 | .25 | .25 | .19 | .14 | .09 |
| | .42 | .30 | .47 | .47 | .40 | .30 | .19 | .09 |
| | .72 | .32 | .54 | .57 | .47 | .28 | .17 | .11 |
| Nitrite | .12 | .13 | .18 | .17 | .16 | .13 | .08 | .03 |
| | .39 | .40 | .40 | .39 | .29 | .16 | .10 | .03 |
| | .70 | .60 | .62 | .50 | .39 | .14 | .06 | .04 |

ties. The verification of this assumption and many of the other aspects of manipulated tone rendering must be left to future treatment.

The type 3 sulfide density-dilution curves of Fig. 4 require careful interpretation. When a bleached silver bromide image is immersed in a sulfide solution containing a silver solvent, the lightest densities, say below .15, darken to the usual extent but the heavier densities, notably those above .5, turn to a sickly transparent yellow, and in a step wedge the upper steps often show reversal. If this wedge is now given the fullest intensification in sulfite, all the upper densities darken to a deep reddish brown and the reversal, so far as we have been able to penetrate the steps, is cured. With incomplete intensification, which is necessary to study these dilution effects, the densities above .5, having been reduced to about .2 in sulfiding, may not reach the original .5 during intensification although they may have acquired as much as .25 density units of metallic silver. The over-all intensification remains negative although the increment of silver during the exposure stage is positive. The density increase, shown with respect to the original silver image in Fig. 4, is repeated, therefore, with respect to the bleached image, in Fig. 6.

Two items should be noted. The general form of the curves, distorted in Fig. 4, comes into line with the others in Fig. 6, and the optimum solution concentration is well marked. Then, again, the activity of the densities is reversed—those below .1 scarcely intensifying, while the heavier steps (within limits not accurately determined but probably .1–.7) acquire density superproportionally.

FIG. 6.




This suggests a new factor in the situation—the concentration of the deposit in the emulsion. For the type 3 sulfide to intensify, it appears necessary for it to be present in a definite concentration in the emulsion. One milligram dispersed in unit area of gelatin of *.x* mm. depth will intensify, where the same quantity in a layer 2 times *.x* mm. deep remains

inactive. It would seem that in the sulfite-type 3 reaction, metallic silver is not the primary light product but is generated secondarily from the decomposition of a reducing material formed at the sulfide surface. It is well known that hydrosulfite [5] and some of the thionates [6] are destroyed by sulfite and if some such intermediate product were involved, there may be a balance between the rate of formation and the rate of destruction of the intermediate. The quantity surviving at any moment will be inversely proportional to the volume in which the sulfide is dispersed, and perhaps a critical survival concentration will be required to yield reduced silver.

We have remarked that a type 3 sulfide image will not intensify in a nitrite solution. In its preparation, the ammonia and hypo have been added to the sulfiding bath solely to disperse or smudge the image grains. More may have been accomplished, however, and the yellow type 3 image may be an allotropic form of sulfide, a polysulfide,[8] or any variety of sulfide with an adsorbed polythionate layer. To test this assumption, a dispersed wedge image of type 3 sulfide was prepared and then bleached and resulfided in pure freshly prepared sodium sulfide solution. This image, physically of type 3 but chemically of type 1, was cut into strips and exposed, one in nitrite and one in sulfite solution. The nitrite, formerly inactive, was now very effective, and an image, intensified far beyond the possibilities of a type 1 deposit, was obtained. In the sulfite solution good intensification took place but the final deposit was black instead of the characteristic type 3 sulfite reddish brown and was of not quite the same depth. The densitometric data are recorded in Table III, and the corresponding curves in Fig. 7. We may fairly conclude that the type 3 image is built up on a polysulfide-polythionate base.

#### THE RELATION BETWEEN EXPOSURE AND GAMMA.

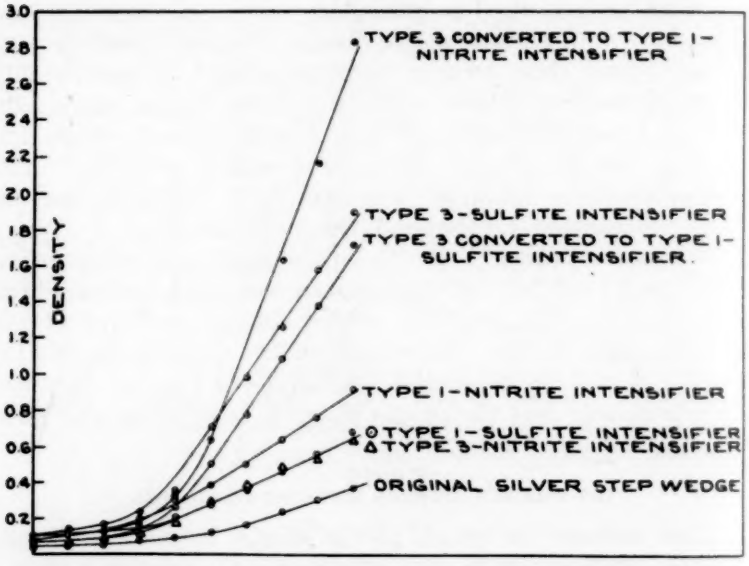
The manner in which the densities increase with the progress of intensification resembles the way gamma increases with time for ordinary alkaline development. The type 1 sulfide images in sulfite or nitrite solution give slightly different $\gamma$-$t$ curves from the type 2 and 3 images in sulfite, since the latter show a small induction period—further evidence of the

TABLE III.

*Showing the Effect on Intensification of Bleaching Type 3 Sulfide Images and Resulfiding in a Type 1 Bath. Par Speed Portrait Film, 4 Minutes, 6 Inches from 250-Watt Lamp.*

| Original Silver Wedge. | Sulfite Intensifier. | | | Nitrite Intensifier. | | |
|---|---|---|---|---|---|---|
| | Type 1 Sulfiding. | Type 3 Sulfiding. | Type 3 Sulfiding Converted to Type 1. | Type 1 Sulfiding. | Type 3 Sulfiding. | Type 3 Sulfiding Converted to Type 1. |
| $d = .04$ | .10 | .06 | .10 | 1.0 | .08 | .08 |
| .05 | .12 | .09 | .16 | .12 | .09 | .13 |
| .07 | .14 | .14 | .23 | .19 | .12 | .18 |
| .09 | .20 | .35 | .26 | .30 | .19 | .32 |
| .12 | .28 | .71 | .50 | .38 | .29 | .64 |
| .16 | .35 | .99 | .78 | .50 | .38 | .98 |
| .23 | .46 | 1.26 | 1.09 | .64 | .49 | 1.64 |
| .30 | .55 | 1.58 | 1.38 | .76 | .54 | 2.16 |
| .37 | .68 | 1.90 | 1.72 | .92 | .65 | 2.82 |

FIG. 7.




formation of an intermediate product before silver deposits on a type 3 image.

A family of characteristic curves for increasing time of intensification were obtained for a set of type 1 wedges ex-

posed in nitrite solution; and the growth of the individual densities taken from the same data is shown in the intensification-time curves of Fig. 8. A type 1 image in sulfite yielded the intensification-time curves of Fig. 9. Similar data, for type 3 sulfided wedges, are shown in Figs. 10 and 11, the former displaying the characteristic curves, the latter the growth of the densities.

#### INTENSIFICATION AND TIME OF ORIGINAL DEVELOPMENT.

Nitrite solutions on the three types of sulfide images and sulfite on type 1 sulfide, yield during a given exposure a fairly constant density increase, irrespective of the events which led to the production of the primary density. For example, let two film areas be exposed and developed to *similar density*, one by unit exposure and development to unit gamma, and one by two units of exposure and development to a gamma of .5, and let both, after sulfiding, be exposed simultaneously in the same intensifying solution. It will be found that the two density increases are similar. When, however, the type 3 sulfiding is employed and the images are intensified in sulfite solution, the original 2-unit exposure will intensify more than the other. The effect on a family of characteristic curves is striking (Fig. 12). All exposures below a threshold density of .1–.15 remain untouched but the higher exposures are increased so that all development times yield the same high gamma and it is the apparent speed of the emulsion which is made to vary with the time of original development.

#### AGITATION DURING EXPOSURE.

The irregularity of the density curves recorded in many of the graphs is due in great measure to the manipulation accorded the densitometric strips. These, as we have said, were exposed in test tubes in the water cell and received no agitation other than the natural convection current flowing up the light warmed strip and down the side of the test tube. When the solutions are agitated with nitrogen bubbles (care being taken not to interfere with the light rays) the densities are more regular and with nitrite, type 1 sulfiding, are a little more intense. With sulfite type 3, the stirred densities are much *less* opaque when the wedge is illuminated from the

FIG. 8.

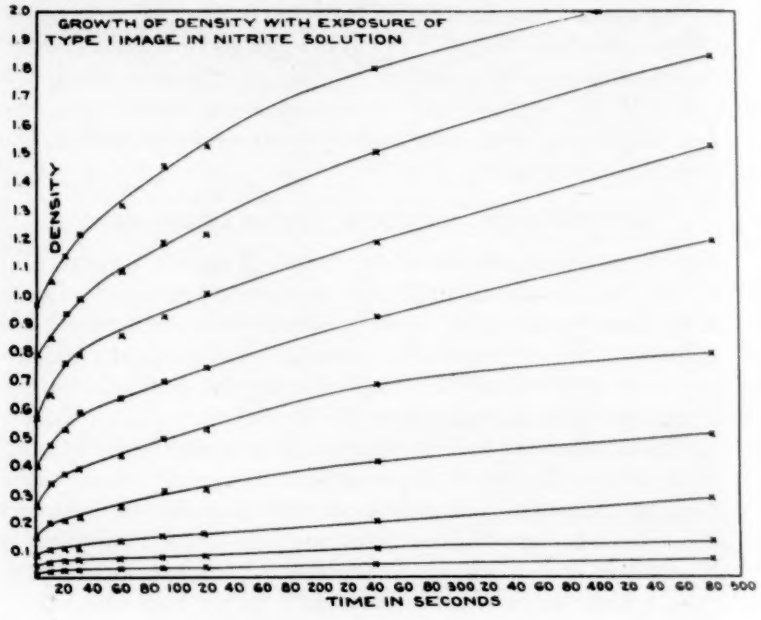

GROWTH OF DENSITY WITH EXPOSURE OF
TYPE I IMAGE IN NITRITE SOLUTION
DENSITY
TIME IN SECONDS

FIG. 9.

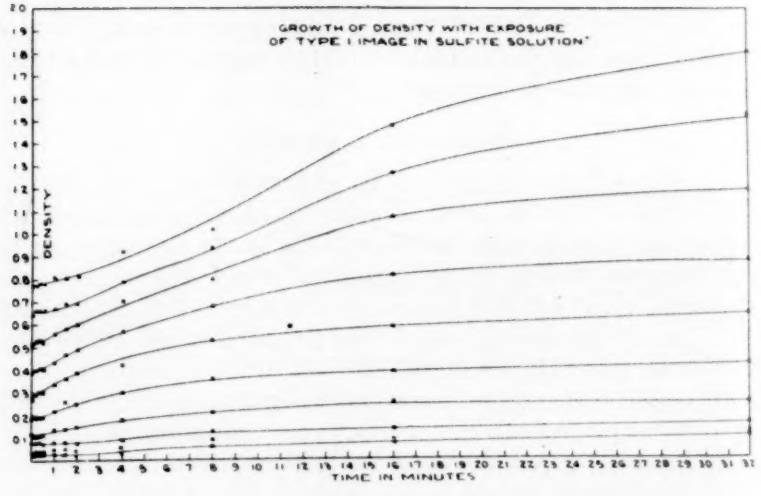

GROWTH OF DENSITY WITH EXPOSURE
OF TYPE I IMAGE IN SULFITE SOLUTION
DENSITY
TIME IN MINUTES

FIG. 10.

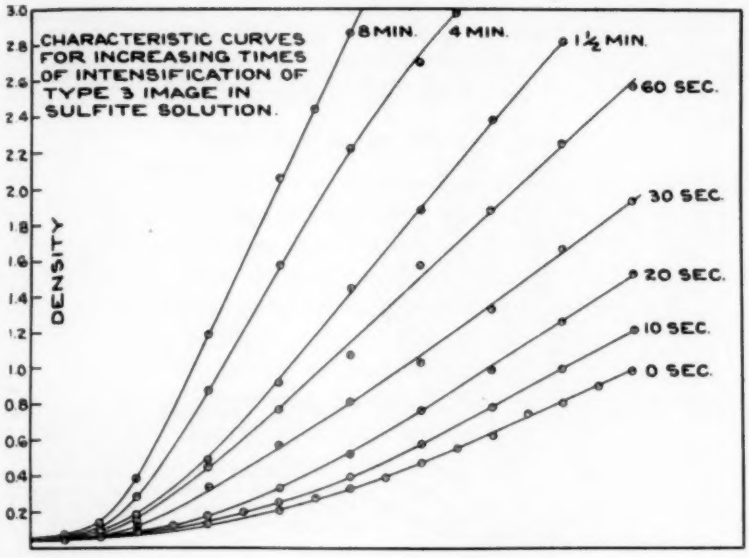

CHARACTERISTIC CURVES FOR INCREASING TIMES OF INTENSIFICATION OF TYPE 3 IMAGE IN SULFITE SOLUTION.
8 MIN.
4 MIN.
1½ MIN.
60 SEC.
30 SEC.
20 SEC.
10 SEC.
0 SEC.
DENSITY
3.0
2.8
2.6
2.4
2.2
2.0
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2

FIG. 11.

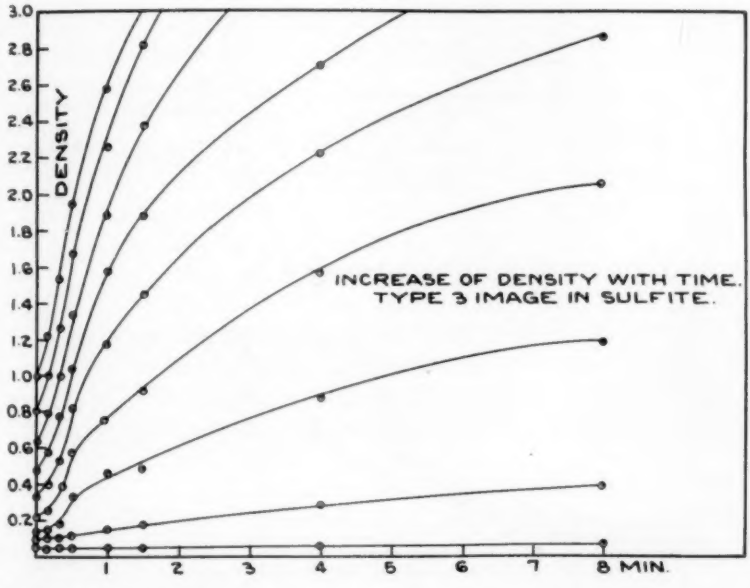

DENSITY
INCREASE OF DENSITY WITH TIME. TYPE 3 IMAGE IN SULFITE.
3.0
2.8
2.6
2.4
2.2
2.0
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
1
2
3
4
5
6
7
8 MIN.

FIG. 12.

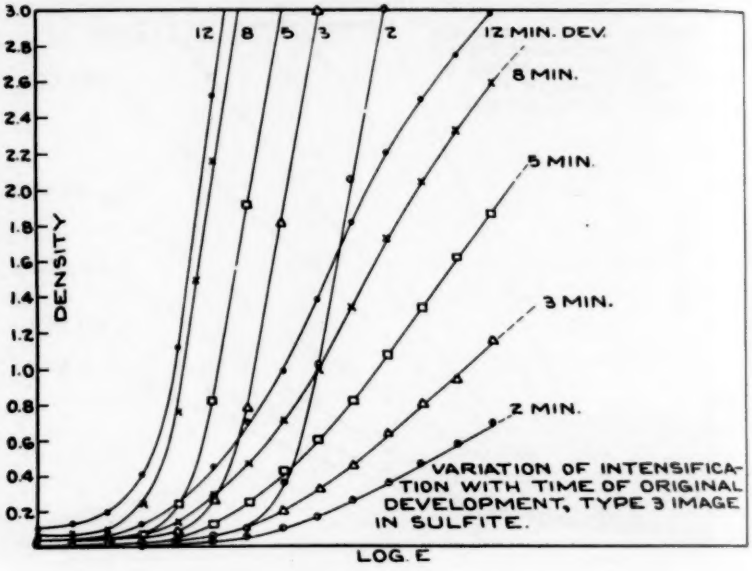
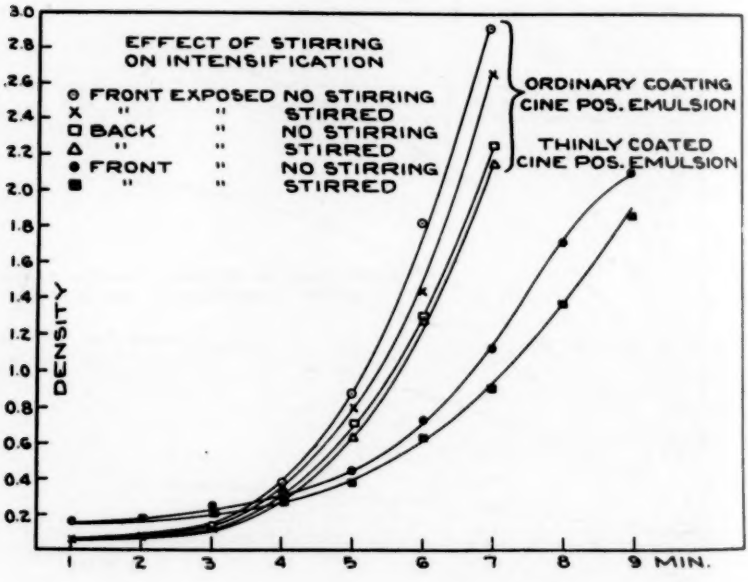

12
8
5
3
2
12 MIN. DEV.
8 MIN.
5 MIN.
3 MIN.
2 MIN.
DENSITY
LOG E
VARIATION OF INTENSIFICATION WITH TIME OF ORIGINAL DEVELOPMENT, TYPE 3 IMAGE IN SULFITE.

FIG. 13.


EFFECT OF STIRRING ON INTENSIFICATION
FRONT EXPOSED NO STIRRING
" " STIRRED
BACK " NO STIRRING
" " STIRRED
FRONT " NO STIRRING
" " STIRRED
ORDINARY COATING CINE POS. EMULSION
THINLY COATED CINE POS. EMULSION
DENSITY
MIN.

front, slightly less opaque when illuminated from behind. This is shown in Fig. 13. Again we have evidence that an intermediate product is needed for the deposition of silver, and this is being swept from the film by agitation.

### THE COLOR OF THE EXPOSING LIGHT.

The color of the silver sulfide deposits suggests that they are more transparent to infra-red than to blue light. Since the building up of the heavy densities must depend largely on the ability of light to penetrate the deposit, it would be expected that exposure to the infra-red would preserve the longer straight characteristic curve. This proves to be the case.

Two broad spectral regions were isolated, one by the use of a liquid cell filled with copper sulfate solution, and one by the infra-red filters [9] lately become popular for long distance photography.* Intensification was quicker and better by infra-red.

The behavior of the two colored lights could be shown graphically, after the manner adopted earlier in this paper, namely, by plotting for comparison, the original characteristic curve of a step wedge and by its side the intensified curves for blue and red light. Perhaps a more useful presentation is that offered in Figs. 14 and 15, where the densities after intensification are plotted against those (Ag—not $Ag_2S$) beforehand. The steepness of the curve at any part is then a measure of the intensification. In all six pairs shown, the infra-red action is preferable from the point of view of tone rendering. The nitrite solution exerts its greatest action in the lighter densities and can practically remove the toe of the characteristic curve. In Fig. 16, curve 28, for nitrite—type 2 in infra-red light, is compared with a curve secured by doubling † the densities of the unintensified wedge. Not only are the significant toe densities added to the straight line portion in the intensified curve but the apparent *H* and *D* speed, as usually computed from the intersection of the straight line extension and the axis, is increased some four times.

* For the purpose of intensification the light of an unscreened tungsten lamp duplicates the specially isolated infra-red.

† To demonstrate a perfectly proportional intensification of 2.

FIG. 14.

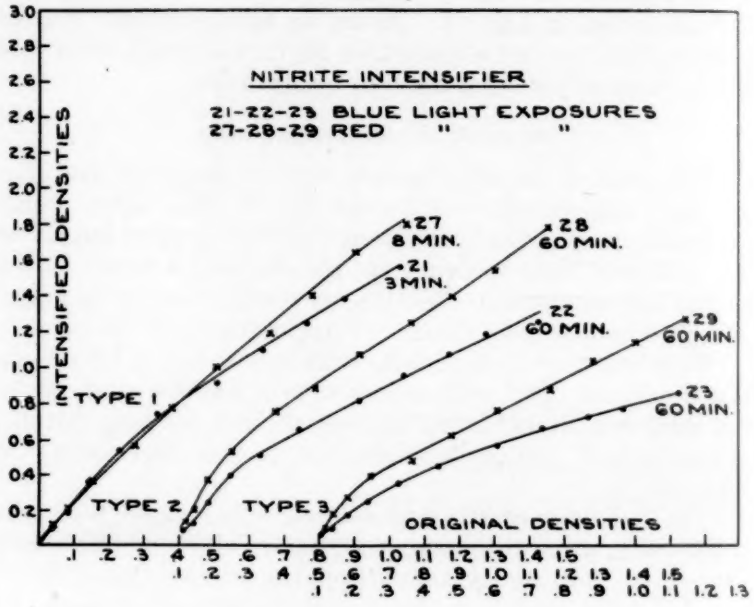

NITRITE INTENSIFIER
21-22-23 BLUE LIGHT EXPOSURES
27-28-29 RED " "
INTENSIFIED DENSITIES
ORIGINAL DENSITIES
TYPE 1
TYPE 2
TYPE 3
Z7 8 MIN.
Z8 60 MIN.
Z1 3 MIN.
Z2 60 MIN.
Z9 60 MIN.
Z3 60 MIN.

FIG. 15.

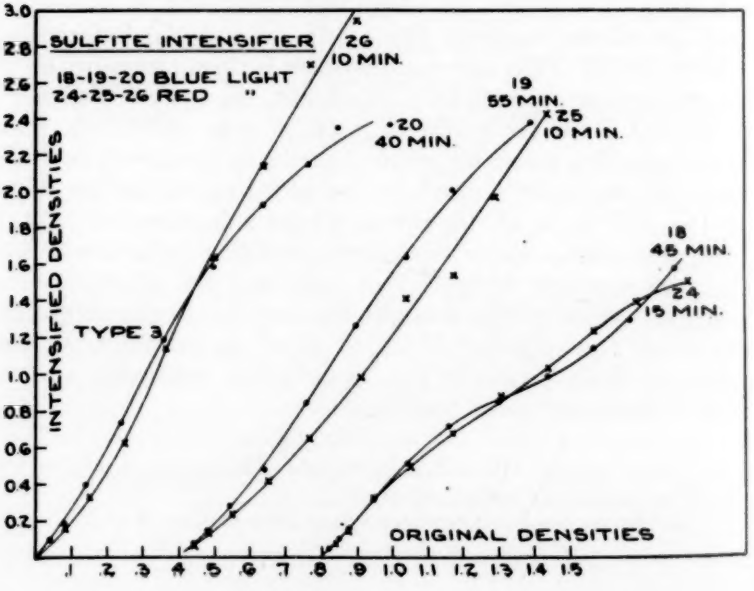
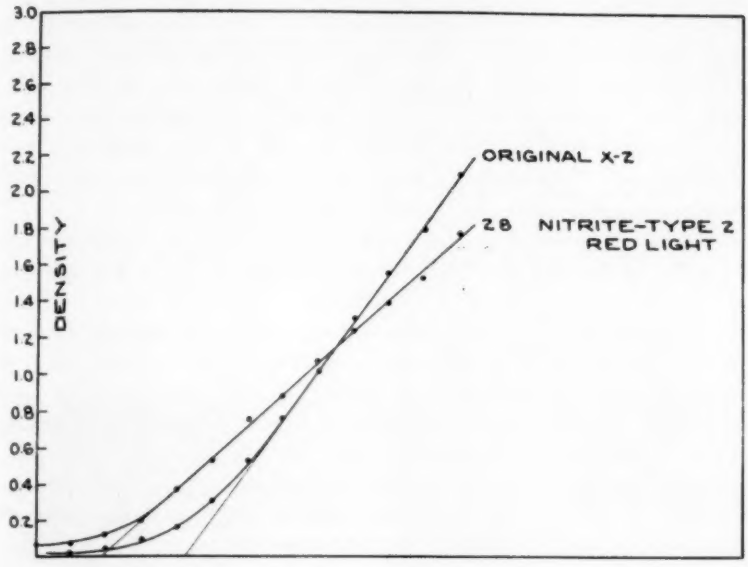

SULFITE INTENSIFIER
18-19-20 BLUE LIGHT
24-25-26 RED "
INTENSIFIED DENSITIES
ORIGINAL DENSITIES
TYPE 3
Z6 10 MIN.
Z0 40 MIN.
19 55 MIN.
Z5 10 MIN.
18 45 MIN.
Z4 15 MIN.

FIG. 16.




The data with infra-red and blue light were secured with intensifying solutions of optimum concentration; with weaker mixtures the toe of the curves would have been lifted in the case of the sulfite also. Space does not permit further comment.

### RESOLVING POWER, DEFINITION AND GRAININESS.

The grains in the plain sulfided images (type 1) grow during intensification and it is to their increase in diameter that the added density is due. Insofar, then, as grain size determines graininess, the process increases graininess. However, graininess depends much more on clumping than mere grain size, and clumping has not been found to increase appreciably with either nitrite or sulfite treatment of type 1 images.

The smudged grain of types 2 and 3 sulfiding presents a different problem. Definition is impaired in direct proportion to the smudging—the resolving power for type 3 images on motion picture positive film being reduced to about one-half. The reduction on coarse grained negative material is correspondingly less. The graininess of type 3 images on the other

hand is enormously increased. After sulfiding, each grain may be considered diminished in size, by erosion, but increased in effective area by the presence of an aura of dispersed sulfide. This tenuous material is affected only when present in a sufficient concentration and the outer fringes of isolated grains are ineffective. The fringes of neighboring grains reinforce one another by overlapping, and thus a clump may intensify where an isolated grain remains scarcely altered.

In Fig. 17, Section *A* is a photomicrograph from a medium density of developed high speed emulsion. "*B*" is from a similar density after type 1 sulfiding, "*C*" after type 3 sulfiding. "*D*" and "*E*" are the corresponding intensified images and the grain clumping in "*E*" is well marked. X-ray images intensified to the limit by the type 3 method acquire a "pepper and salt" appearance visible some distance from the naked eye. This objectionable feature may be offset by the phenomenal gain in density, which, in one particular experiment enabled a film which had received one-sixteenth the normal exposure to be brought to the same tonal condition as a correctly exposed comparison negative. It is hoped to apply the method to the quantitative estimation of graininess.

## CHEMICAL CONSIDERATIONS.

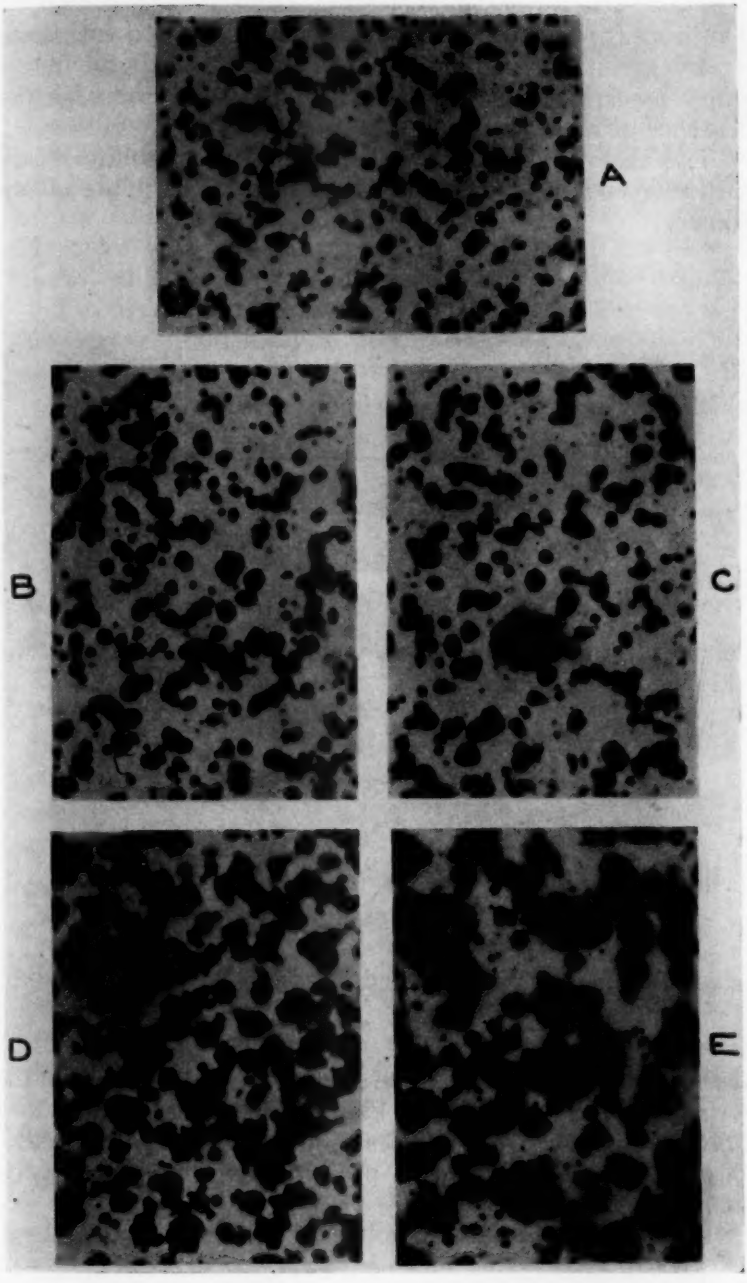
### Some Interesting Solutions; the Mechanism of the Reaction.

A silver sulfide image will darken more or less when illuminated under many reducing solutions containing free silver ion: Fig. 18 shows the growth of type 1 sulfide image in a representative set of solutions for a certain moderate exposure. It is apparent that an enormous field for densitometric study could be opened up if the properties of these various solutions were recorded for the many kinds of negative and positive materials, each subjected to variations in bleaching and sulfiding. The preliminary evidence, however, is that sulfite and nitrite solutions are the most powerful and useful, though their chemical mechanism is different.*

The intensification properties of sodium-silver-nitrite solu-

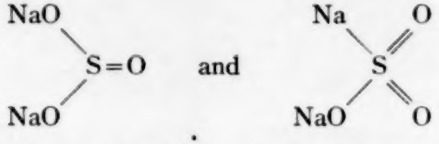
* The intensifying action comprises two parts—the reception of light energy by the silver sulfide and its transmission to the solution as an ionic change; and the utilization by the solution of this change. The first part is probably common to all the intensifiers, the latter varies specifically with each mixture.

FIG. 17.

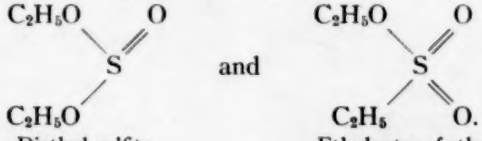


tions appear to be well defined and can be recorded with some certainty; not so these of sodium silver sulfite. Is the uncertainty associated with impurities in the sulfite or with the existence of more than one form of sulfite?

The chemical structure of "pure" sodium sulfite is still open to question,[10] but there is the admitted possibility of two isomers:



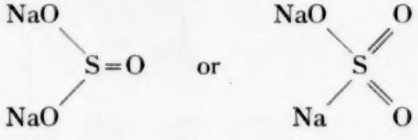

which have their counterparts in the organic



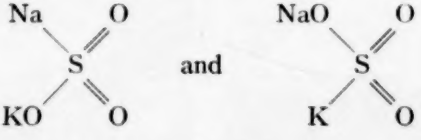

Diethyl sulfite

Ethyl ester of ethyl sulfonic acid

On hydrolysis, both materials yield identical sodium sulfite, and evidence of such salts as




or the mixed salts



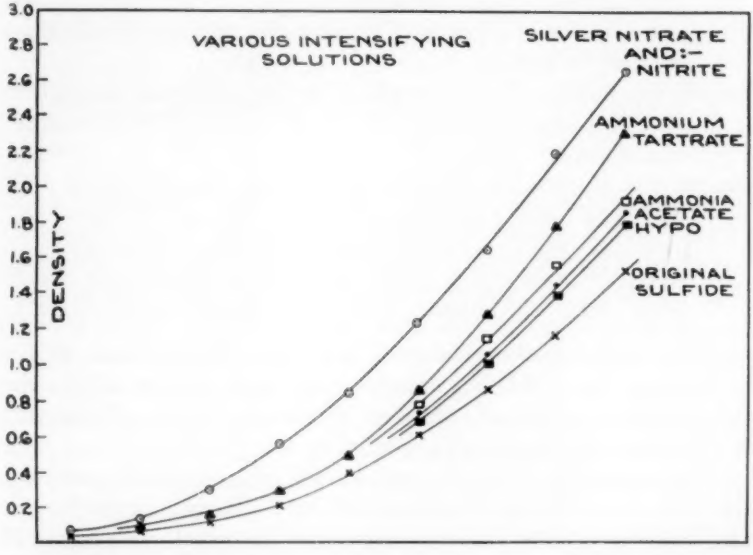

has never been secured.[11] The evidence is thus in favor of the normal existence of only one kind of inorganic sulfite. Now intensifiers made from sulfites from different sources differ so greatly in activity, that presumably some impurity is modifying their behavior.

It is known that sulfite solutions on long aging undergo

auto-reduction [12] with the deposition of sulfur and the production of sulfates or sulfuric acid. In intermediate stages the sulfur and sulfite are evidently in equilibrium with some thiosulfate and consequently with the thionates. If any free acid is present part of the sulfite will be in the meta form $SO_3.SO_2^=$.[13] A silver ion on the surface of activated silver sulfide would thus have a wide choice of sulfurous ions and molecules with which to react and again the question obtrudes, which of these impurities confers the sensitivity?

The first experimental requirement is a sample of pure sulfite—say, sodium sulfite. The most promising source is not necessarily from the neutralization of pure NaOH solution

FIG. 18.




with sulfur dioxide gas, since at the mixing interface some of the solution must become acid and the metabisulfite may be formed. It is thought better to use this "pure" sulfite as a preliminary step and to prepare from it, by fractional precipitation, the insoluble sulfite of a metal which yields more soluble thiosulfates, sulfates and thionates, so that the latter will be rejected on washing. A second double decomposition

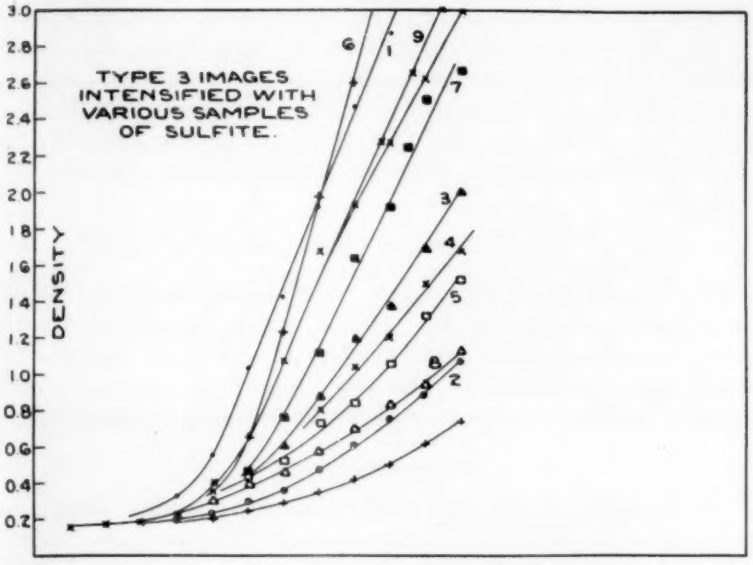
can then give the sodium salt. Strontium sulfite meets the requirements fairly well, and it is from strontium sulfite decomposed with sodium carbonate that the purest sodium salt has been prepared. The following samples of sulfite were reserved for test:

*Sample 1.* Pure sodium sulfite (Baker's "Analyzed").
*Sample 2.* Eastman Kodak "Photographic."
*Sample 3.* Same as sample 1, recrystallized just before use.
*Sample 4.* Sample 1 is precipitated with strontium chloride, the precipitate washed with sodium carbonate, and filtered. The filtrate is estimated by iodine titration and used immediately.
*Sample 5.* The sulfite is prepared from $SO_2$ gas (cylinder) and (*a*) sodium carbonate solution, (*b*) sodium hydroxide solution, taking care to keep the product at all times slightly alkaline.
*Sample 6.* Sulfite from sample 5 is treated with strontium chloride and the *middle fraction* extracted with sodium carbonate.
*Sample 7.* Pure sodium hydroxide solution is saturated with $SO_2$ gas until colored a bright yellow and is then neutralized with more sodium hydroxide. This yields a mixture containing about 4 parts of $Na_2SO_3$ to one of $NaHSO_3$. It is used directly without crystallization.

With these materials and a number of additional sulfur derivatives the following experiments were done to trace the effect on the intensified characteristic curve. The effects may be discussed in conjunction with Fig. 19.

The sulfided curve (type 3) for the original material (thinly coated Motion Picture Positive) is shown as the lowest line. Curve 1 is after intensification in standard intensifier (5 grams $AgNO_3$ per liter) using sample 2 sulfite. This curve represents great intensification, is easily reproducible and alters little with different batches of commercial sulfite. It is the curve which is strictly comparable with the majority of others in this paper. Curve 2 is from intensifier compounded with strontium-derived sulfite. The solution is scarcely active, and at first some doubt was felt that the strontium salt had been properly decomposed by the carbonate, but the

FIG. 19.




iodine titration together with the ready re-solution of the first white silver sulfite precipitate in excess of the reagent confirmed the presence of the proper quantity of soluble sulfite. If now the solution just mentioned is acidified with acetic acid, the curve (3) is secured, showing an increased sensitivity, which scarcely diminishes when the solution is again made alkaline (curve 4).

Sample 5 sulfite made from $SO_2$ and excess alkali was a little more active (curve 5) than the pure sample 4 sulfite, but sulfite made with an excess of $SO_2$ (sample 7) gave the steepest intensification curve of all (curve 6). The pure sample 4 sulfite was then contaminated with small quantities of tetrathionate yielding curve 7, and hypo giving curve 8. Dithionate was without effect and trithionate was as good a promoter as tetrathionate. Some Wackenroder's solution, which is reputed to contain all the thionates, was made by bubbling $H_2S$ through sulfurous acid and was added to intensifier made with pure sulfite. The brown $Ag_2S$ precipitate was removed and the filtrate used to provide curve 9. The most powerful promoter of all was made by saturating dilute

hypo with $SO_2$. Such a solution is said to contain quantities of tri- and pentathionates in equilibrium with colloidal sulfur.

At this stage of the argument we may voice the vague opinion that the promoting substance is "available" sulfur. For the sulfur to be unsaturated with oxygen seems insufficient since pure sulfite and the dithionates have low activity; all those substances which promote intensification contain the linkages:

$$S'' = S^{VI} \qquad S^{VI} - S'' - S^{VI}, \qquad \text{or} \qquad S^{VI} = S^{IV}.$$

We have compared the many coördination formulæ which have been proposed for tri-, tetra-, and pentathionates with those for metabisulfite and thiosulfate, and there appears no significant uniformity.[12] The only property in common seems to be an ability to present sulfur to a suitable acceptor (Ag).

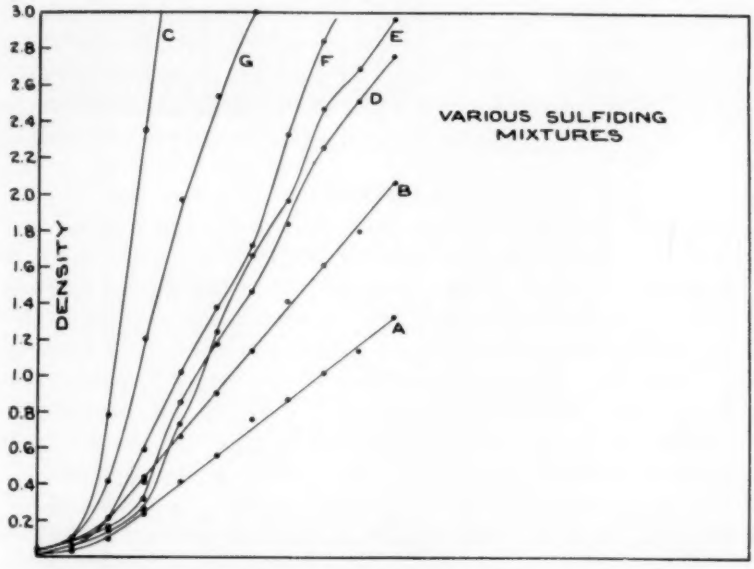
### THE SULFIDING BATH.

It is unlikely that chemically pure silver sulfide, that is to say, a material accurately represented by the symbols $Ag_2S$ is ever produced from the simple sulfide toning of a silver image. The sulfide is more likely to be $Ag_2S_{(1\times x)}$ where $x$ is a small but widely variable fraction of additional sulfur. This sulfur may be present as a proportion of true polysulfide, or as adsorbed thiosulfate, polythionate, or even as elementary sulfur. All the sulfide images we have examined show an instantaneous darkening when treated with dilute silver nitrate, the change having nothing to do with optical intensification as such. The darkening may amount to only .01 density unit in a heavy deposit, but it occurs. It is the least marked in images produced from fresh, specially pure sodium sulfide. Stale sulfide and polysulfide solutions yield images which nearly double in density on immersion in silver nitrate. High contrast developing-out papers which yield unsatisfactory yellow tones, even in fresh sulfide baths, show marked darkening and improvement in tone with silver nitrate. Evidently fine grained deposits retain a larger proportion of available sulfur than coarse-grained ones.

Alterations to the dispersity of the image by altering the sulfide bath inevitably do more than diminish the particle size—they increase the ability to retain sulfur and they may

contribute new varieties of absorbable material. Thus, the inclusion of hypo in the sodium sulfide bath greatly reduces the grain size by solution of the silver halide and precipitation of silver sulfide in the interstices between the grains and it correspondingly increases the light response of the image. But ammonia, in similar quantities, which has only a fraction of the solvent power on the halide grains, and gives little dispersion, nevertheless gives great sensitivity; and a mixture of ammonia, sodium sulfide and hypo promotes intensification to the greatest degree and in a manner not to be expected from the effect of the two additions separately. While, therefore, a certain argument led to experiments with silver solvents in the

FIG. 20.




sulfide bath, and while the experiments yielded the desired result, it is certain that the premises were only partly right. Bearing this in mind, the curves in Fig. 20 may be inspected. *A* is the original silver wedge which was then cut into strips, bleached in permanganate bromide, and intensified in silver sulfite after sulfiding in various ways. Of these *B* represents pure .3 per cent. $Na_2S$ and *C*, the steepest curve ever ob-

TABLE IV.

*Per Cent. Composition of Sulfiding Bath.*

| Curve Number. | Figure Number. | $Na_2S$. | $Na_2S_2O_3$. | KSCN. | $NH_4OH$ .880. | KBr. | KI. | Remarks. |
|---|---|---|---|---|---|---|---|---|
| A | 20 | | | | | | | Original silver wedge |
| B | 20 | .3% | | | | | | |
| C | 20 | .1% | .5% | | .5% | | | Very great intensification |
| D | 20 | 1.0% | 3.0% | | | | 2% | Poor tone reproduction |
| E | 20 | 3.0% | 5.0% | | | | | Poor tone reproduction |
| F | 20 | .03% | .05% | | | | | Too long immersion in sulfide bath required. |
| G | 20 | .3% | .5% | | | | | Good sulfiding mixture |
| 1 | 21 | .1% | .5% | 1% | | | | Numbers 1, 2, 8 and 6 reject varying numbers of low density steps, above which, however, intensification is good. The very high densities are all uneven |
| 2 | 21 | .2% | .5% | 1% | | | | |
| 3 | 21 | .3% | .500 | 1% | | | | |
| 5 | 21 | .3% | .2% | 1% | | | | |
| 8 | 21 | .3% | 2.0% | | | | 1% | |
| 11 | 21 | .3% | 1.0% | 1% | | 2% | | Excellent intensification from the lowest densities upwards |
| 4 | 22 | .3% | | | | | | Original silver wedge |
| 5 | | .1% | | | 1% | | | These solutions all give good straight and steep curves, the unevenness being due to lack of stirring. They provide a choice of solutions which begin to operate above varying fog densities |
| 7 | | .2% | | | 3% | | | |
| 9 | | .2% | | 1% | 3% | | | |
| 10 | First in | .1% | | | 2% | | | |
| | and then in | .3% | | | 3% | 1% | | |

tained after 10 minutes' illumination, was produced from .1 per cent. $Na_2S$ containing .5 percent. hypo and .5 per cent. of .880 ammonia.

The wavy curves, *D* and *E*, were procured with sulfiding baths containing too large a concentration of sulfide and solvent, and *F* was derived from a solution too dilute. *G* has the same hypo content as *C*, but the sulfide is stronger and the intensification correspondingly less.

Other variations in the sulfiding bath are recorded in Table IV and in Figs. 21 and 22, from which their characteristics may be derived by inspection.

### TEMPERATURE COEFFICIENT AND RECIPROCITY FAILURE.

Measurements of the temperature coefficients of the intensification processes; the variations of sensitiveness with pH; and the response to different intensities and colors of light under conditions of constant exposure ($I \times t$) merits a degree of precision not attempted in this general survey. It is hoped to devote a separate paper to such work.

### SUMMARY.

Photographic images of silver sulfide immersed in solutions of silver salts and exposed to light increase in density. The reaction can be made the basis of a kind of physical intensification provisionally named "Optical Intensification." Solutions of silver-sodium-nitrite and silver-sodium-sulfite, containing about 5 gms. silver per liter, are most effective. The reaction in nitrite is a simple oxidation-reduction process which causes metallic silver to deposit on the silver sulfide.

The reaction in sulfite may occur in a manner analogous to the nitrite or it may proceed through the medium of adsorbed poly-sulfur-bodies, in which case the generation of a diffusible reducing material is thought to precede the deposition of silver.

The silver sulfide images in all the solutions examined have been found to be panchromatic, the sensitiveness extending from the violet into the infra-red.

By variation in the chemical procedure and in the color and intensity of the exposing light, the tone rendering of the intensified image may be controlled within wide limits. On the

FIG. 21.

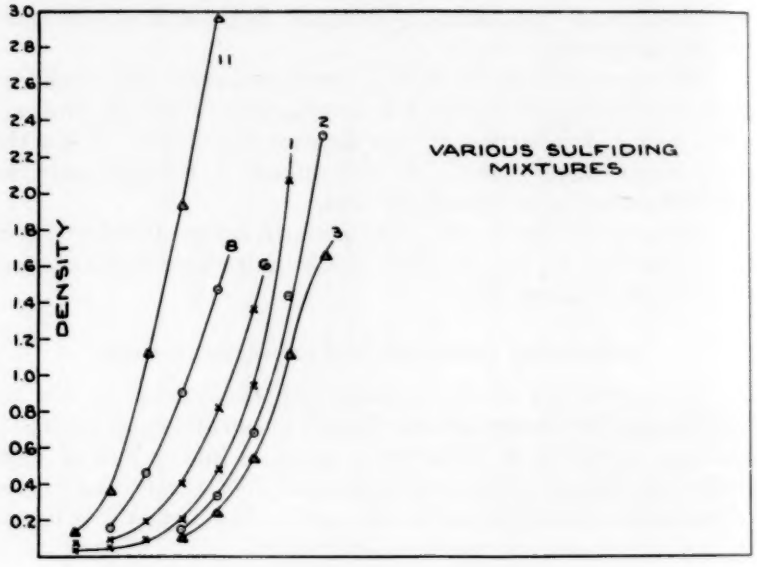
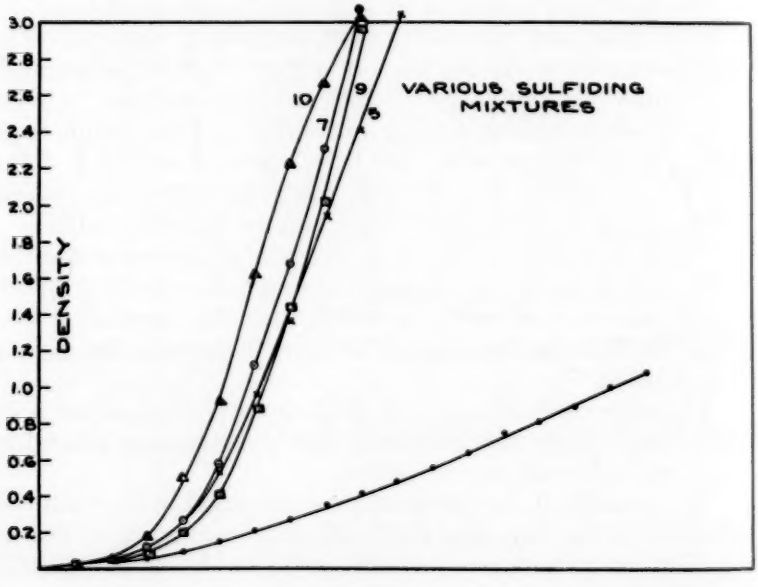

DENSITY
VARIOUS SULFIDING MIXTURES
11
2
1
8
6
3

FIG. 22.


DENSITY
VARIOUS SULFIDING MIXTURES
10
9
7
5

one hand fog, and the lower densities may be neglected entirely while the significant densities are emphasized; on the other, the toe of the characteristic curve may be lifted until it becomes substantially continuous with the straight line portion.

**REFERENCES.**


1. HICKMAN, K., AND WEYERTS, W.: *J. Frank. Inst.* (December, 1933).
2. See: Slater Price, T.: *Phot. J.*, **62**, 356–367 (1922), for many references.
3. FAJANS, K., AND FRANKENBURGER, W.: *Z. physik. Chem.*, **105**, 255 and 273 (1923).
   EGGERT AND NODDACK: *Z. Physik.*, **20**, 299 (1923); *Z. Physik.*, **31**, 922 (1925).
   See also: Sheppard, S. E., and Vanselow, W.: *J. Phys. Chem.*, **33**, 250 (1929).
   IMHOFF, A.: Beiträge zür Kenntnis der Lichtempfindlichkeit der Silberhaloide. Dissertation, Giessen, 1928.
4. LÜPPO-CRAMER: *Phot. Ind.*, 982 (1924).
5. JELLINEK, K.: "Das Hydrosulfit," Ferdinand Enke, Stuttgart (1911).
6. MACKENZIE, J. E., AND MARSHALL, H.: *J. Chem. Soc.*, **98**, 1726 (1908).
7. HICKMAN, K., AND WEYERTS, W.: *Brit. J. Phot.*, **80**, 482–484 (1933).
8. KNOX, J.: *Trans. Faraday Soc.*, **4**, 48 (1908).
   DITTE, A.: *Comptes rend.*, **120**, 91 (1895).
   LUMIÈRE, A. AND L., AND SEYEWETZ, A.: *Rev. franç. phot.*, **4**, 133 (1923).
9. Wratten filter No. 87, see descriptive article. W. Clark, *Amer. Phot.*, **26**, 675 (1932).
10. MELLOR, J. W.: "Inorganic and Theoretical Chemistry," Volume X., S and Se—, pp. 235–241.
11. SABANEJEEF, A.: *Z. anorg. allge. Chem.*, **17**, 480 (1898);
    FRAPS, G. S.: *American Chemical Journal*, **23**, 202 (1900).
12. LEWIS, G. N., RANDALL, M., AND BICHOWSKY, F. R. V.: *J. Amer. Chem. Soc.*, **40**, 356 (1918).
13. EPHRAIM, F. E., AND AELLIG, C.: *Helv. Chim. Acta*, **6**, 37 (1923).
14. RIESENFELD, E. H., AND FELD, G. W.: *Z. anorg. allge. Chem.*, **119**, 225 (1921).
    MARTIN, F., AND METZ, L.: *Z. anorg. allge. Chem.*, **127**, 83 (1923).

**Amylase of Human Saliva.**—Amylase is the enzyme which hydrolyzes starch through the various dextrins to maltose. Its presence in the saliva has been known for over a century. W. LLOYD ADAMS AND VICTOR C. MYERS (*Jour. Dental Res.*, 1933, XIII, 311–322) have made an elaborate study of the amylolytic index of human saliva. This index may be defined as the per cent. of the starch which is hydrolyzed to reducing sugar when 0.01 cc. of saliva and 0.01 gram of soluble potato starch are incubated at a temperature of 40° C. for 30 minutes at a total dilution of 1 : 200. While the index ranged between 25 and 50 in a group of 26 normal persons, yet it remained unusually constant in a given individual; thus, during the course of 5 months, its maximum variation in one person was ± 2 units. The index was markedly influenced by the chloride and total phosphorus content of the saliva; an increase of either 79 per cent. in the chloride content or of 43 per cent. in the total phosphorus content was accompanied by an increase of 45 per cent. in the amylolytic index. The index decreased as much as 50 per cent. in persons with severe "colds," possibly as the result of excessive secretion of saliva, and tended to be high in persons susceptible to dental cavies.

J. S. H.

**Bottled Juices.**—D. C. CARPENTER (*Ind. and Eng. Chem.*, Ind., Ed., 1933, XXV, 932–934) have studied the influence of light upon sterile apple juice and kraut juice stored in white glass bottles. The red end of the spectrum darkens the color, while the blue end tends to fade the color of the juice. With apple juice, the red end improves the aroma and flavor, the blue end decreases both. With kraut juice, the red end changes the aroma and flavor and tends to produce the characteristic aroma of decaying cabbage, while the blue end tends to produce a sour, saline taste; yellow, blue-green, and blue light increase the turbidity; green light gives results next best to storage in the dark. For marketing beverage juices, they should be placed in containers of green glass, or else the clear white glass container should be wrapped in a transparent green cellulose covering.

J. S. H.

# ELECTRICAL RESISTANCE AND HEAT IN METALS.

BY

CHARLES R. UNDERHILL,

Consulting Electrical Engineer.

Just what is electrical resistance? It is defined by the laws of Ohm and Joule but has different bases in various media. The positive resistance of a cold-cathode vacuum tube is dependent upon the energy required to force the electrons from the cathode and that consumed in their constant acceleration until they strike the anode, as well as the number of electrons in the space between the cathode and the anode with which it inversely varies. The negative resistance in the gaseous discharge or arc depends upon the numbers of electrons and positive ions present. In electrolytic conduction, wherein there also are negative and positive currents flowing in opposite directions, the electrolyte may be thought of as consisting of two conducting channels, one for the negative and one for the positive ions, the sum of the two ionic current strengths being the total current strength.

Considering only the negative or the positive current strength,

$$I = \frac{Q}{L}v = \frac{q}{l}v \qquad \text{amp.,} \quad (1)$$

wherein $Q/L$ is the number of coulombs ($6.281 \times 10^{18}$ monovalent ions per coulomb) per cm., $q$ and $l$ are the electric and linear displacements, and $v$ is the ionic velocity, the ratio of the potential gradient to the ionic velocity is constant at any given temperature for each kind of ion in any kind of aqueous solution, whence

$$\frac{E}{Lv} = \frac{RQ}{L^2} \qquad (2)$$

is a constant quantity at any given temperature, showing that the resistance gradient varies inversely as the number of coulombs per linear cm.

Most metals, unlike gases with negative resistances and liquids with positive resistances and negative temperature coefficients of resistance, have both positive resistances and temperature coefficients, the resistance increasing as heat is stored in the metal. Yet the classical dimensions of resistance are the same for all of the above conditions and are based upon the mechanical force electrically developed between two point charges. It is evident, therefore, that if the mean velocity $v_m$ is substituted for $v$ in equation (2) that equation should hold for all media, the resistance gradient varying inversely as the number of coulombs per cm. at any given temperature. In what follows, the number of atoms per cm. turns out to be of the general order of the number of electrons in a negative coulomb.

From the electrothermal energy equation is derived

$$Wt = q^2R \qquad \text{joule-sec.,} \quad (3)$$

wherein $R$ is the action per quantity of electricity squared and from which it follows that the resistance in ohms is numerically equal to the action in joule-sec. when $q$ is unity.

Let a direct current flow in a thermally insulated solid metal conductor to increase its temperature from o to $T$ deg. K., assuming that the resistance is substantially equal to zero at o deg. K. Then the resistance of the metal also will increase and the total heat stored in the conductor at any temperature $T$ deg. K. will be

$$W_t = m\int c_v dT = c_m mT = \frac{m}{A}\int C_v dT = \frac{m}{A}C_m T \quad \text{joules} \quad (4)$$

and

$$W_t = \frac{q^2}{tT}\int RdT = \frac{q^2}{t}R_m \qquad \text{joules} \quad (5)$$

will be the corresponding total expenditure of electrical energy during said storage, wherein $c_m$ and $C_m$ respectively refer to the mean specific heats in joules per gram and gram atom.

Then

$$\frac{q^2}{t}\frac{R_m}{C_m T} = \frac{m}{A} \quad \text{gram/gram atom.} \quad (6)$$

When $q^2$ is made numerically equal to $t$, as when 1 amp. flows for 1 sec. to store the heat and increase the resistance, $I^2 = 1/t$ and $R_m$ becomes numerically equal to $W_t$ although their classical dimensions are different. Then, since $m/A$ is constant at all temperatures, $R_m = \frac{m}{A} C_m T$ at all temperatures. $I$ is the mean or effective current strength in amperes.

This is found substantially the case for Cu and Fe by plotting the I. C. T. percentages of the 0 deg. C. resistances at various lower temperatures *vs.* absolute temperature, the smaller values being plotted to larger scales, and the areas under the smooth curves found by planimeter. A similar procedure gives the values of $C_m T$.

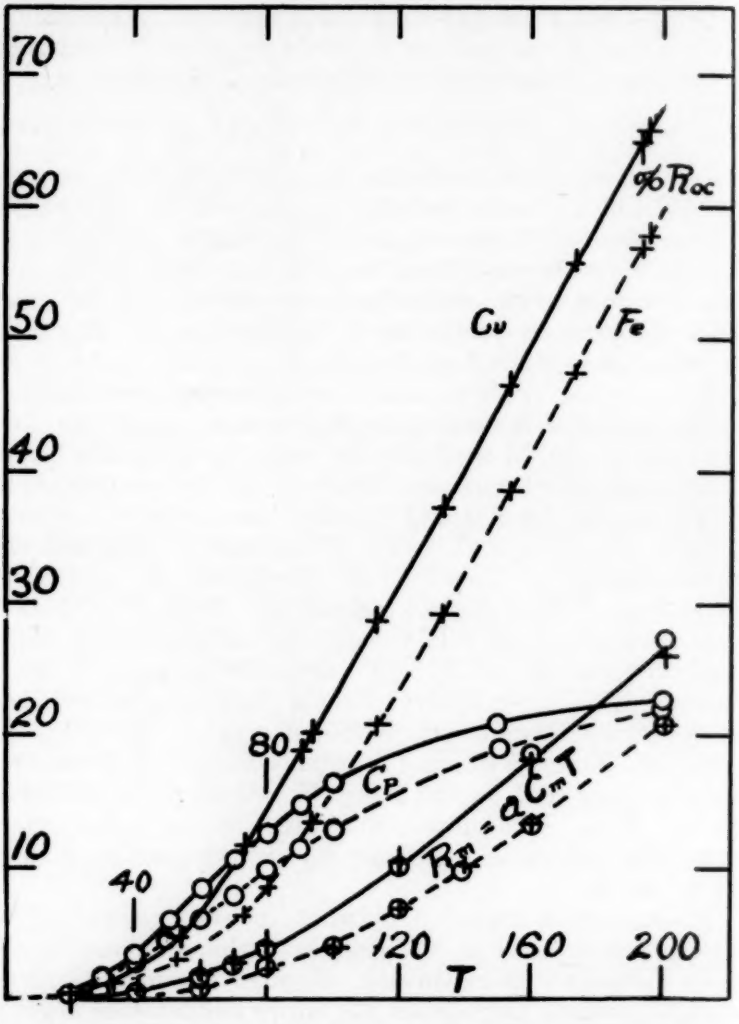
Because specific heat data even at normal temperatures seldom are more accurate than 1 per cent., resistivity data vary, and graphical methods are inexact, the results herein obtained are considered satisfactory under the circumstances.

The graphs for Cu and Fe show the respective plotted I. C. T. percentages of 0 deg. C. resistance $\%R_{oc}$ and the specific heats $C_p$. The $R_m - T$ curves also are plotted as percentages of the 0 deg. C. resistances. In the $a\ C_m\ T - T$ curves, $a$ is a constant used to make the ordinates numerically equal to those in the $R_m - T$ curves for purposes of comparison. The $R_m - T$ curve for Cu shows that at 200 deg. K. the mean resistance $R_m$ is 26 per cent. of the 0 deg. C. resistance, while the limiting or total resistance $R_t$ at 200 deg. K. is 67.5 per cent. of the 0 deg. C. resistance, as indicated by the $\%R_{oc} - T$ curve. Reference to the latter curve shows that the mean resistance $R_m$ (26 per cent.) occurs at 113.5 deg. K.

Expansion is neglected in what follows because of the evident inaccuracies above noted and the smallness of the mean coefficients of expansion. Because the mass of metal remains constant throughout the entire temperature interval from 0 deg. K. to the chosen limiting temperature $T$, the resistance varying at constant mass, it follows that the number of atoms, as well as the number of atoms per layer and the number of layers of atoms, remains constant at all temperatures for any kind of metal and specific value of $q^2/t$.

At the temperature of mean resistance, the conductor

FIG. 1.




first is assumed to be a centimeter cube of solid metal having a mass numerically equal to its density and a mean volume resistivity $\rho_m$ which is taken as the product of the indicated percentage of the volume resistivity at o deg. C. and that resistivity. Then the cross-section which will cause the equality in equation (6) is found.

$$\frac{q^2}{t} R_m = \frac{q^2}{t} \frac{L}{S} \rho_m = \frac{m}{A} C_m T = \frac{d}{A} LSC_m T \quad \text{joules,} \quad (7)$$

whence

$$S = \sqrt{\frac{q^2}{t} \frac{\rho_m}{C_m T} \frac{A}{d}} = \sqrt{\frac{q^2}{t} \frac{\rho_m}{C_m} \frac{V_a}{T}} \quad \text{cm.}^2. \quad (8)$$

The resistance and mass gradients are

$$\frac{R_m}{L} = \frac{\rho_m}{S} \quad \text{ohm/cm.} \quad (9)$$

and

$$\frac{m}{L} = Sd \quad \text{gram/cm.} \quad (10)$$

The number of atoms per linear cm. is

$$\frac{N}{L} = N_0 \frac{S}{V_a} = N_0 \sqrt{\frac{q^2}{t} \frac{\rho_m}{V_a C_m T}} \quad \text{atoms/cm.,} \quad (11)$$

which holds for all temperatures.

Referring to the table, it will be understood that the calculated values of $S$ and the three following quantities dependent upon $S$ vary many per cent. from their average values in these computations.

The number of atoms per cm. is of the general order of the number of electrons in a negative coulomb when $q^2 = t$, which also is true for other metals.

By making $q^2 = t$, since then $I^2 = 1/t$ and

$$T = \frac{\left(\frac{T}{t}\right)}{I^2} \quad \text{deg.,} \quad (12)$$

TABLE.

*Values for Cu and Fe at 200 deg. K.*
*($q^2 = t$.)*

| | Cu. | Fe. |
|---|---|---|
| $\rho_m$ | $4.12 \times 10^{27}$ | $1.77 \times 10^{-6}$ ohm-cm. |
| $C_m T$ | $2.72 \times 10^{3}$ | $2.33 \times 10^{3}$ joules/gram atom |
| $d$ | 8.89 | 7.86 grams/cc. |
| $V_a$ | 7.15 | 7.10 cc./gram atom |
| $S$ | $3.29 \times 10^{-5}$ | $7.37 \times 10^{-5}$ cm.$^2$ |
| $R_m/L = W_t/L$ | $1.25 \times 10^{-2}$ | $2.40 \times 10^{-2}$ ohm/cm. |
| $m/L$ | $2.92 \times 10^{-4}$ | $5.80 \times 10^{-4}$ gram/cm. |
| $N/L$ | $2.80 \times 10^{18}$ | $6.30 \times 10^{18}$ atoms/cm. |

$t$ becomes the time-limit during which the storage of heat $W_t$ and the change in resistance $R_t$ occur over a corresponding temperature interval beginning at 0 deg. K. and ending at any predetermined temperature $T$ deg. K., the required difference of potential varying with $R_m$. Hence if $q = 2$ coulombs, 4 sec. are required for the temperature to increase to 200 deg., for example, and $T/t = 50$ deg./sec., whereas if $q = 1$ coulomb, only 1 sec. is required for the same increase in temperature and $T/t = 200$ deg./sec.

Thus the variable current flows for a definite period and then stops, the temperature increase always being numerically equal to the limiting temperature when $q^2 = t$, that is, when $R_m$ is made numerically equal to $W_t$, but $q^2$ and $t$ may have any magnitudes whatever so long as they are numerically equal. If a smaller limiting temperature is chosen, the mean resistance and the constant difference of potential will be smaller, but the above equality equally applies to all limiting temperatures.

The conditions for $q^2 = t = 1$, $R_m = W_t$, and $T/t = T$, are when the current density is about $3.04 \times 10^4$ amp./cm.$^2$ for Cu and about $1.36 \times 10^4$ amp./cm.$^2$ for Fe.

It would appear that a refinement of the method herein outlined might lead to a general equation for temperature coefficients of resistance at all temperatures up to near the melting points of metals, the first step being the determination of the equation to the $R_m - T$ curve.

# THE DIELECTRIC PROPERTIES OF CHLORINATED DIPHENYLS.

BY

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Bell Telephone Laboratories.

As a part of a general study of the relation between dielectric properties and both chemical and physical structure the chlorinated diphenyls are of considerable interest because of their chemical relationship to the chlorinated naphthalenes, which are widely used in condensers on account of their relatively high dielectric constant associated with low dielectric loss. Dielectric theory postulates certain definite changes of dielectric constant and dielectric loss with frequency which depend in magnitude, and the frequency at which they occur, upon the polarizable constituents of the material. One of the physical concepts advanced to explain dielectric behavior is the orientation of polar molecules suggested by Debye.[1] Dielectrics containing polar constituents should show a region of anomalous dispersion and the location of this region on the frequency scale, according to Debye, is a function of the molecular size and the viscosity, which of course changes with temperature.

The chemically pure individuals of the chlorinated diphenyl series are not readily available and furthermore those that have been prepared are crystalline and, in this state, have low dielectric constants. The small values usually observed and the difficulty of getting sufficiently good contact for accurate dielectric measurements on crystalline solids make them difficult to study. However, certain commercial mixtures of chlorinated diphenyls are obtainable which range from a mobile oil to brittle resins. Since all of these mixtures presumably contain polar constituents this wide range of physical properties furnishes an excellent opportunity to establish the relationship between viscosity and anomalous dispersion in a new and distinctive type of material.

[1] Polar Molecules, P. Debye, Chemical Catalog Co., 1929.

The materials used in this investigation were commercial chlorinated diphenyls [2] differing in chlorine content and presumably in the position of the substituted chlorine atoms and having the physical properties listed in Table I. They were measured as received without any attempt at purification and presumably represent the commercial product.

TABLE I.

*Physical Properties of Chlorinated Diphenyls.*

| No. | Condition (25°). | Condition (0°). | Pour Point. | Distilling Range. |
|---|---|---|---|---|
| 1... | Mobile Oil | Viscous Liquid | −17.7° to −20.7° | 311°–356° |
| 2... | Viscous Oil | Sticky Resin | 8° to 12.6° | 352°–381° |
| 3... | Sticky Resin | Brittle Resin | 34.9° to 38.0° | 373°–404° |
| 4... | Brittle Resin | Brittle Resin | | |
| 5... | Crystalline Solid | Crystalline Solid | | |

The dielectric measurements consisted of grounded capacitance and conductance measurements using a capacitance bridge described by Shackelton and Ferguson [3] and a test condenser consisting of three concentric, gold-plated brass cylinders whose air capacity was about 65 mmf. This condenser was sealed in a glass tube which was immersed in a thermostat maintained at constant temperature ± 0.1° C.

Following a practice which is coming into wide use at present, the dielectric properties of these materials are expressed in terms of the real and imaginary parts of the complex dielectric constant. The total dielectric constant is given by the equation:

$$\epsilon = \epsilon' - i\epsilon'',$$

where $\epsilon'$ represents the real part or ordinary dielectric constant and $\epsilon''$ the imaginary part of the dielectric constant, or dielectric absorption. $\epsilon''$ is sometimes called the loss factor also. These quantities and their relation to the tangent of the loss angle, which is commonly used for expressing dielectric loss, are defined as follows:

[2] These products were supplied through the courtesy of the Swann Chemical Company, their producers. They cover the range of commercially available chlorinated diphenyls known as Aroclors.

[3] Shackelton and Ferguson, *Bell System Tech. J.*, **7**, 70 (1928).

$$\epsilon' = \frac{C}{C_0},$$

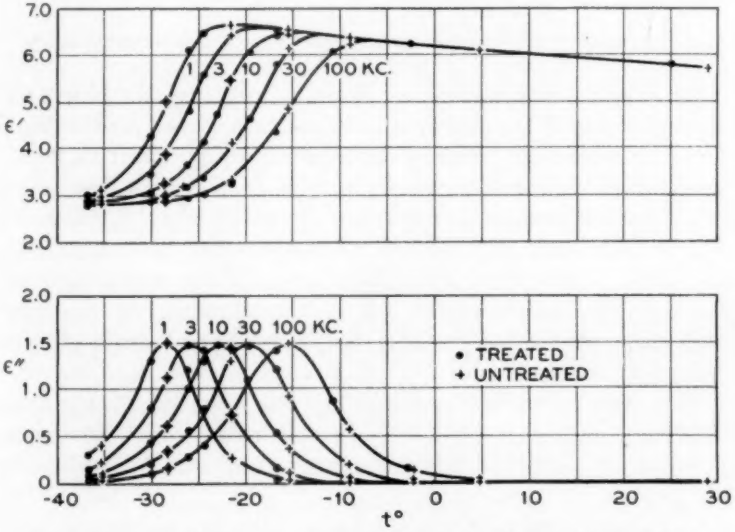
$$\epsilon'' = \frac{G}{2\pi f C_0},$$

$$\tan \delta = \frac{\epsilon''}{\epsilon'},$$

where $C$ is the capacity of the condenser filled with the dielectric, $C_0$ the capacity of the empty condenser, $G$ the equivalent parallel conductance of the dielectric, and $f$ the frequency of the applied field. No correction for direct current conductance was made in this work since it was negligibly small, except at the higher temperatures. The principal purpose of the work was to determine the position of the regions of anomalous dispersion and so the measurements are largely confined to the temperatures where anomalous dispersion is present.

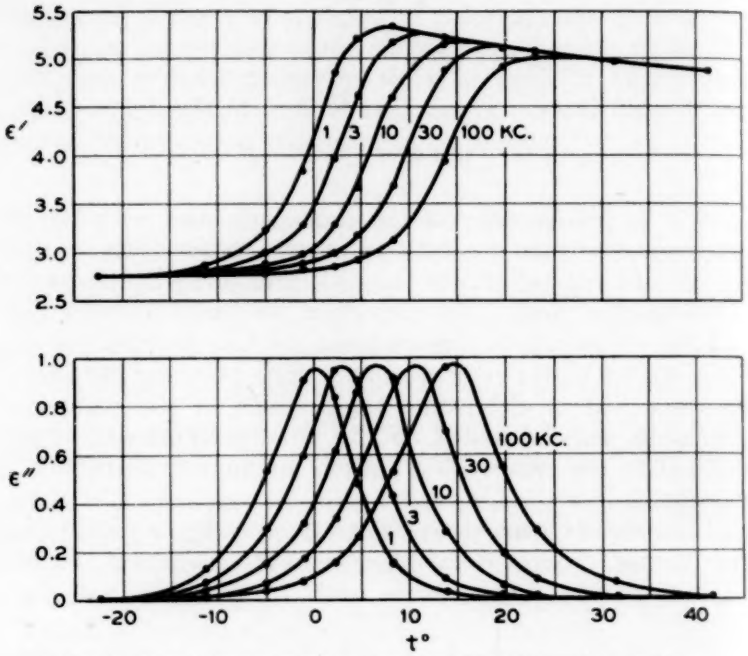
The results obtained are shown graphically in Figs. 1 to 4, and do not require detailed explanation, although several

FIG. 1.




Real and imaginary dielectric constant vs. temperature for chlorinated diphenyl No. 1 treated and untreated.

FIG. 2.



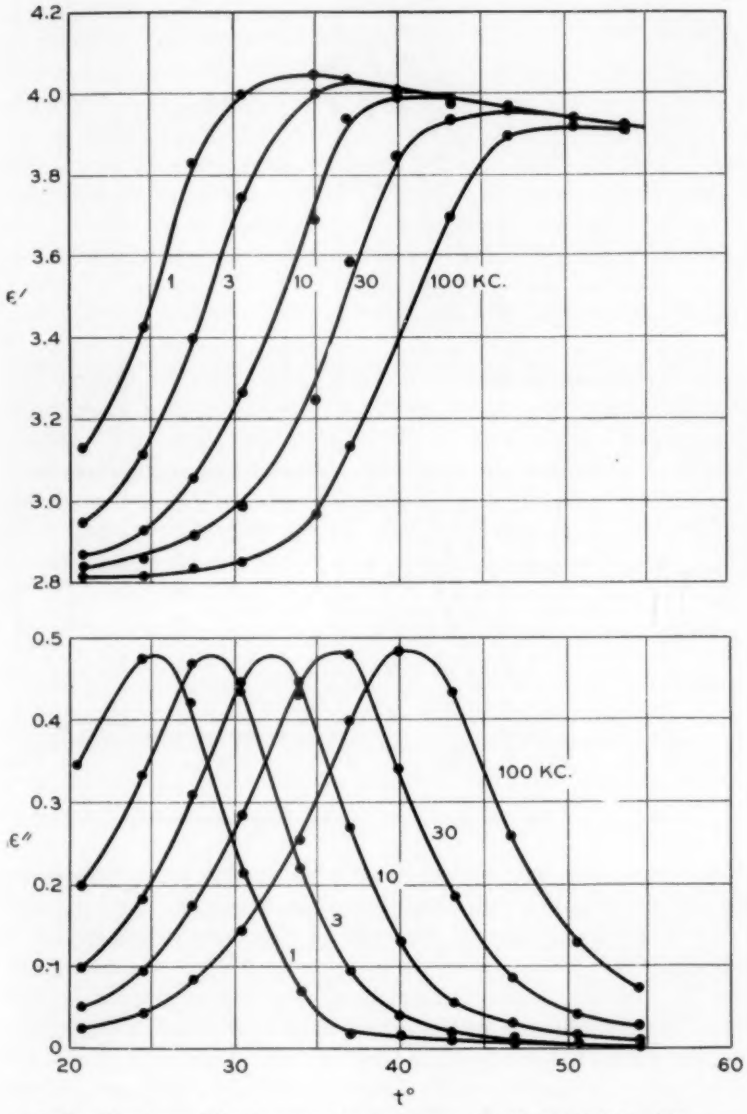

Real and imaginary dielectric constant vs. temperature for chlorinated diphenyl No. 2.

points may be emphasized. Since a mixture of chlorinated diphenyls would be presumed to contain polar molecules, it should be expected to show anomalous dispersion; that is, a change of dielectric constant and loss factor with frequency, at certain temperatures where the viscosity is so high that the natural frequency of rotation of the dipoles becomes of the same order as the alternating field used for the measurements. This behavior is shown by all of the chlorinated diphenyls which form glasses on cooling; No. 5 does not form a glass on cooling but crystalizes.

This behavior is not unusual but has been shown for many insulating materials such as rosin,[4] rosin oils,[5] insulating oils [6] and mixtures of rosin and transformer oils.[7] Anomalous

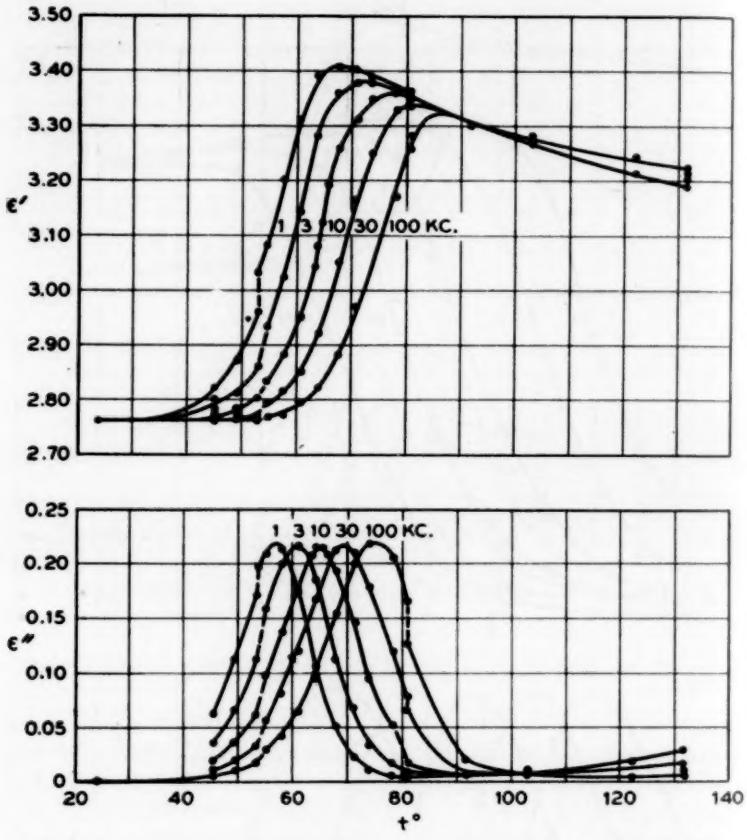
[4] D. W. Kitchen and H. Muller, *Phys. Rev.*, **32**, 979 (1928).
[5] S. O. Morgan and A. H. White, J. FRANK. INST., **213**, 313 (1932).
[6] H. H. Race, *Phys. Rev.*, **37**, 430 (1931).
[7] E. Kirch and W. Riebel, *Arc. f. Elektrot.*, **24**, 355, 533 (1930).

FIG. 3.




Real and imaginary dielectric constant vs. temperature for chlorinated diphenyl No. 3.

FIG. 4.




Real and imaginary dielectric constant vs. temperature for chlorinated diphenyl No. 4.

dispersion has also been shown in many pure substances known to be polar such as glycerine, alcohols [8] and glycols.[9]

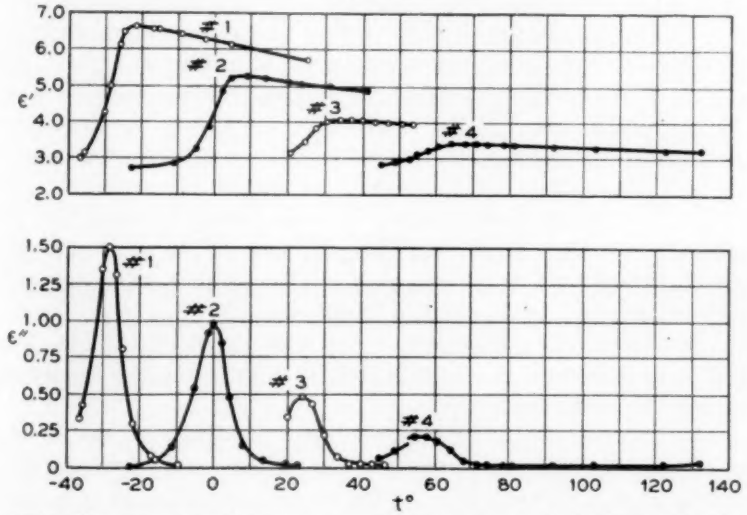
A brief discussion of any one of the curves will suffice to illustrate the behavior of them all. In Fig. 1, for compound No. 1, the dielectric constant, $\epsilon'$, increases almost linearly with decreasing temperature and is independent of frequency, in the range studied, from the highest temperatures which were measured down to about $-5°$ C. Between $-10°$ and

[8] S. Mizushima, *Bull. Chem. Soc. Japan*, 1, 47, 83, 115, 143, 163 (1926).
[9] A. H. White and S. O. Morgan, *Physics*, 2. 313 (1932).

– 40°, however, there is a region of anomalous dispersion and in this region both $\epsilon'$ and $\epsilon''$ change with frequency. $\epsilon'$ decreases from its high value of about 6.5 at temperatures where the viscosity is not great enough to interfere with the orientation to its low value of 2.7, towards which the curves are converging, at temperatures where the viscosity is too great to permit any orientation of polar molecules. Because of the absence of any appreciable ionic conductance at these temperatures it is believed that the loss factor, $\epsilon''$, represents a loss due to polar molecules or aggregates. Starting from a low value at about the same temperature at which $\epsilon'$ begins to change, $\epsilon''$ passes through a maximum after which it again decreases to a low value at temperatures where no further rotation of dipoles is possible.

Entirely analogous behavior is shown by materials numbered 2, 3 and 4 as can be seen from the corresponding figures. The temperatures at which anomalous dispersion is found are different, however, for each material, being higher the greater the viscosity (see Table I). In Fig. 5 the data for $\epsilon'$ and $\epsilon''$, plotted against temperature at one kilocycle, are shown for those materials which form glassy resins. The

FIG. 5.




Real and imaginary dielectric constant vs. temperature at 1 k.c. for the chlorinated diphenyls.

differences between the values of dielectric constant and loss factor, and the different temperature ranges in which anomalous dispersion occurs, are clearly shown in this curve. Chlorinated diphenyl No. 1, which has the highest dielectric constant, also has the largest value of the maximum of $\epsilon''$. However, in all cases, $\epsilon''$ decreases to a very small value at temperatures above and below the region of anomalous dispersion. The conclusion to be drawn is that in order to secure a low loss and high dielectric constant from these materials they must be used at temperatures above that where anomalous dispersion is found. These data do not cover the high temperature range very completely. The curve for No. 4, which extends to higher temperatures, shows a behavior which is typical and which has been pointed out by many observers for many dielectrics; namely, an increase of the loss at higher temperatures due to an increase of ionic conduction.

The curves for No. 4 in Fig. 4 have several features which require explanation. There are breaks shown by dashed lines in both $\epsilon'$ and $\epsilon''$ Curves. These indicate places at which mixture No. 4 was held at 80° C. in the test condenser overnight, necessitated by the impossibility of obtaining all the readings in any one day and the likelihood of damaging the condenser if the material were allowed to solidify completely and then be reheated each day. These breaks indicate a progressive change in the material on standing at 80° C., the change being in the direction of an increase of viscosity. Also, the dielectric constant at temperatures above the region of anomalous dispersion, about 90° in this case, does not appear to be independent of frequency as might be expected, but shows a slightly higher value for one kilocycle than for 100 kilocycles. This is due to the presence of ionic conduction great enough to cause an electrolytic polarization capacity which is larger the lower the frequency.

A considerable change in color was observed for No. 5 on melting, although no significant ill effects on the dielectric properties were observed.

Measurements were made on two samples of No. 1, one untreated and the other treated by the suppliers with Fuller's earth. As shown in Fig. 1, where the solid circles represent data on treated and crosses the data on untreated material, no

difference between these two samples was observed, their curves coinciding.

TABLE II.

*Dielectric Properties of Chlorinated Diphenyls at 25° and 1 K.C.*

| Number. | $\epsilon'$ | $\epsilon''$ |
|---|---|---|
| 1 | 6.70 | 0.008 |
| 2 | 5.05 | 0.005 |
| 3 | 3.48 | 0.48 |
| 4 | 2.76 | 0.008 |
| 5 | 2.51 | 0.007 |

In Table II the dielectric properties of all of these chlorinated diphenyls are given at 25° C. and one kilocycle. Reference to Fig. 5 shows that this temperature does not represent these materials at corresponding states. For Nos. 1 and 2, for example, 25° is above the region of anomalous dispersion and they have relatively high values of dielectric constant and low losses; for No. 3, it is in the region of anomalous dispersion, so that a high loss is found, while for No. 4, 25° is below the region of anomalous dispersion and low values are found for both $\epsilon'$ and $\epsilon''$. In Table III the

TABLE III.

*Dielectric Properties of Chlorinated Diphenyls at Corresponding Temperatures and 1 K.C.*

| Number. | $t°$ | $\epsilon'$ | $\epsilon''$ |
|---|---|---|---|
| 1 | −16 | 6.58 | 0.05 |
| 2 | +13 | 5.24 | 0.04 |
| 3 | 40 | 4.01 | 0.02 |
| 4 | 78 | 3.37 | 0.01 |

dielectric properties are given at what may be called corresponding temperatures; that is, temperatures just above those at which $\epsilon'$ begins to decrease. It should be noticed that these temperatures correspond very closely to the pour points given in Table I.

On the theoretical side, it is believed that dipole polarization best accounts for the observed anomalous dispersion, even though the maximum for $\epsilon''$ is in each case lower than the value predicted by the Debye theory. This might be expected in view of the fact that these chlorinated diphenyls

are mixtures of different molecular species, each with its own relaxation time and polarity.

Apparent relaxation times at various temperatures were calculated for No. 2. Extrapolaration from these values indicated a relaxation time, $\tau$, at 40° of about $1.6 \times 10^{-9}$. The viscosity $\eta$ reported for this material in the descriptive literature of the producers is the equivalent of 4.3 poises at 40°. Calculating average molecular radius, $a$, by the equation $a^3 = kT\tau/4\pi\eta$, where $k = 1.37 \times 10^{-16}$, gives a value of $1.1 \times 10^{-8}$ cm. which is somewhat low, as is usually observed for polar liquids.

It is clear from this work that the temperatures at which these materials may be used in order to take advantage of the high dielectric constant and low loss will depend upon the frequency, and may be readily determined from the individual curves of Figs. 1 to 4. The temperature limit for $10^6$ cycles may be readily extrapolated from the data and found to be about 10° higher than for $10^5$ cycles. Similarly the temperature limit for 100 cycles is found to be about 10° lower than for 1000 cycles. The lower the temperature at which the material is used without reaching the region of anomalous dispersion, the higher the dielectric constant. For No. 1, for example, the dielectric constant reaches a value of about 7 at $-25°$ C., the temperature at which anomalous dispersion is just beginning for power frequencies.

## NOTES FROM THE U. S. BUREAU OF STANDARDS.*

### COMPOSITION AND THERMAL EXPANSION OF SOME SODA-LIME-SILICA GLASSES.

Data were given in Technical News Bulletin no. 176 (December 1931) from which one can compute, approximately, the "critical" and softening temperatures and also the total linear expansion to those temperatures of 33 soda-lime-silica glasses. Since that report 16 additional glasses were made and their expansions were determined by the interferometer method, as in the previous case. The compositions of the new glasses were in the same range as those first studied—that is, the silica content varied from about 52 to 80 per cent., the balance being either soda or soda and lime, but the lime did not exceed 16 per cent.

A simultaneous solution of all the data obtained on the 49 glasses shows that the linear expansions of any of these glasses, expressed in microns per centimeter, can be computed with reasonable accuracy (the maximum variation being 2.5 microns) from

$$E = aA + bB + cC,$$

in which $E$ is the total linear expansion for any temperature range between 25° and 475° C.; $A$, $B$, and $C$ represent the percentages of silica, soda and lime, respectively; and $a$, $b$, and $c$ vary with temperature as follows:

$$a = 0.000222t,$$
$$b = 0.002826t + 0.00000387t^2,$$
$$c = 0.001422t,$$

in which $t$ equals $T - 25°$, $T$ being the temperature at which expansion is computed, and is in general less than 475° C.

### THERMAL EXPANSION OF VITREOUS ENAMELS.

The thermal expansion characteristics of vitreous enamels are commonly expressed in terms of coefficient of expansion.

* Communicated by the Director.

It is a well known fact that the temperature-expansion curves of enamels are not straight lines, and hence that the coefficient of expansion for a given enamel varies according to the temperature range in question.

Data obtained with the interferometer on a group of commercial type ground coats of systematically varying composition indicate that the expansion characteristics of these enamels may be expressed in terms of two constants which are applicable throughout the entire range from room temperature to critical temperature, or any part of it. Thus: $E = a(T_1 - T_0) = b(T_1^2 - T_0^2)$ where $E$ is expansion per unit length, $T_1$ and $T_0$ are the limits of the temperature range involved, and $a$ and $b$ are constants characteristic of the enamel.

In an illustrative case, when $T$ is expressed in °C., $a = 8.35 \times 10^{-2}$, and $b \times 7.19 \times 10^{-5}$. $E$ is then the linear expansion in microns per centimeter between the temperatures $T_0$ and $T_1$. The mean coefficient of linear expansion in a given temperature range may, of course, be obtained by dividing $E$ by $(T_1 - T_0)$.

The $a$ constant represents the "instantaneous" rate of expansion at 0° C., and the $b$ constant represents the acceleration in expansion with respect to temperature. The formula rests upon the finding that this acceleration is a constant for each of the enamels studied.

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## DAMPNESS ON MASONRY WALLS.

As mentioned in Technical News Bulletin no. 197 (September 1933), a letter circular (LC–391) has been prepared on this subject and copies are now ready for distribution.

This letter circular enumerates the causes of dampness on the interior surfaces of masonry walls above grade. The conditions favorable to condensation on masonry walls are described. Inadequate protection of masonry from water collected on roofs or other horizontal or sloping surfaces is cited as a frequent cause of dampness. Relations between rainfall, wind velocities, and absorptive properties of masonry walls are found to indicate that occasional dampness should be expected when plaster is applied directly to the interior

surface of ordinary walls 8 inches in thickness. Different types of colorless waterproofings are compared and methods for treating masonry are discussed. Specific methods are given for remedying dampness in masonry walls.

### TESTS OF IMPORTED REFINED SUGAR AT CHICAGO.

Importations of refined sugars from Cuba are being received at a number of the inland ports. Chicago is receiving regular shipments of considerable magnitude. To facilitate the accurate testing of these sugars at the Chicago Customs laboratory, the Bureau has tested and delivered a new Bureau of Standards type saccharimeter to that port. A member of the Bureau's staff visited the Chicago laboratory in connection with this work and at his suggestion exchange samples of the refined sugar are being received for control test.

### RECOMMENDED COMMERCIAL STANDARD FOR MARKING GOLD FILLED AND ROLLED GOLD PLATE ARTICLES OTHER THAN WATCH CASES.

A general conference held on September 21, 1933, at the United States Chamber of Commerce adopted a recommended Commercial Standard for marking all gold filled and rolled gold plate articles other than watch cases as a basis for promoting fair competition; to enable the consumer to make a direct comparison of quality of gold filled and rolled plate articles; to establish a definite minimum quality for gold filled; to promote the sale of items of real quality; and to establish a better understanding between buyer and seller of gold covered articles.

According to the standard, no articles may be marked gold filled or rolled gold plate unless they are produced by soldering, brazing, welding, or affixing by other mechanical means, a sheet or shell of karat gold upon base metal, said sheet or shell of gold to be rolled, drawn, or pressed to the marked weight ratio before uniting with the base metal.

The recommended standard further requires that articles made by the above process shall be marked "gold filled" only when the quality is 1/20 10K or better. For example, "1/10 12K Gold Filled" means that the article consists of a

base metal covered on one or more surfaces with a gold alloy of 12 karat fineness throughout the gold covering, said covering of gold alloy comprising 1/10th part by weight of the entire article exclusive of joints, catches, screws, and other exemptions recognized by the trade.

In the case of "rolled gold plate" articles, the standard requires that articles marked "rolled gold plate" shall also be marked to show the proportional weight of the coating and the fineness designation.

In addition to the quality mark, it is further required that articles so made and so marked shall have applied thereto a registered trade mark or name of the manufacturer in order to fix responsibility for the quality mark.

The recommended commercial standard has been circulated to producers, distributors, and consumers for written acceptance, and if accepted by a satisfactory majority of production it will, in the absence of active opposition, be made effective for new production on January 1, 1934.

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## NEW AND REVISED PUBLICATIONS ISSUED DURING SEPTEMBER 1933.

### JOURNAL OF RESEARCH.[1]

*Bureau of Standards Journal of Research*, vol. 11, no. 3, September 1933 (RP Nos. 593 to 599, inclusive). Price, 25 cents. Obtainable by subscription.

### RESEARCH PAPERS.[1]

(Reprints from June and July 1933 Bureau of Standards Journal of Research)

RP561. The compressometer, an instrument for evaluating the thickness, compressibility, and compressional resilience of textiles and similar materials; H. F. Schiefer. Price, 5 cents.

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[1] Send orders for publications under this heading only to the Superintendent of Documents, Government Printing Office, Washington, D. C. Subscription to Technical News Bulletin, 50 cents per year; *Journal of Research*, $2.50 per year (United States and its possessions; Canada, Cuba, Mexico, Newfoundland, and Republic of Panama); other countries, 70 cents and $3.25, respectively.

RP562. The thermal expansion of refractories to 1,800° C.; R. A. Heindl. Price, 5 cents.

RP563. Fire tests of columns protected with gypsum; N. D. Mitchell. Price, 5 cents.

RP564. The infrared arc spectra of manganese and rhenium; W. F. Meggers. Price, 5 cents.

RP566. The viscosity of sulphuric-acid solutions used for battery electrolytes; G. W. Vinal and D. N. Craig. Price, 5 cents.

RP567. Composition of grids for positive plates of storage batteries as a factor influencing the sulphation of negative plates; G. W. Vinal, D. N. Craig, and C. L. Snyder. Price, 5 cents.

RP568. The freezing point of iridium; F. Henning and H. T. Wensel. Price, 5 cents.

RP570. A new description and analysis of the arc spectrum of chlorine; C. C. Kiess. Price, 5 cents.

RP572. Creep and structural stability of nickel-chromium-iron alloys at 1,600° F.; W. A. Tucker and S. E. Sinclair. Price, 5 cents.

RP573. An international comparison of temperature scales between 660° and 1,063° C.; W. F. Roeser, F. H. Schofield, and H. A. Moser. Price, 5 cents.

RP576. A 200-kilocycle piezo oscillator; E. G. Lapham. Price, 5 cents.

RP577. The viscosity of optical glass; W. H. Wadleigh. Price, 5 cents.

RP578. The present status of the standards of thermal radiation maintained by the Bureau of Standards; W. W. Coblentz and R. Stair. Price, 5 cents.

RP580. Equipment for testing current transformers; F. B. Silsbee, R. L. Smith, N. L. Forman, and J. H. Park. Price, 5 cents.

RP581. Phase synchronization in directive antenna arrays with particular application to the radio range beacon; F. G. Kear. Price, 5 cents.

### TECHNICAL NEWS BULLETIN.

Technical News Bulletin no. 197, September 1933. Price 5 cents. Obtainable by subscription.

LETTER CIRCULARS.

It is the intent of the Bureau to distribute single copies of these mimeographed letter circulars on request only to those parties having special interest in the individual letter circular. Economy necessitates limitation in the number of copies issued. It is not the intent to supply parties with a copy of each letter circular issued during the month. Letter circulars are necessarily of a temporary nature designed to answer numerous inquiries on a given subject. Requests should be addressed to the Bureau of Standards, Washington, D. C.

LC389. Publications of the Bureau of Standards on X-rays.
LC390. List of weights and measures offices of the States and District of Columbia.
LC391. Damp masonry walls above grade.

# THE FRANKLIN INSTITUTE.

**(PROCEEDINGS OF THE STATED MEETING HELD WEDNESDAY, OCTOBER 18, 1933.)**

The regular monthly meeting of The Franklin Institute was called to order at eight fifteen on the evening of Wednesday, October eighteenth, by the President of the Institute, Nathan Hayward, who presided.

The meeting was crowded to capacity and several hundred were necessarily turned away.

The President called upon the Secretary for the presentation of the business of the meeting.

The Secretary reported that the minutes of the last regular meeting of the Institute held on May seventeenth had been printed in full in the JOURNAL OF THE FRANKLIN INSTITUTE. He requested the President to announce that the minutes be approved as printed unless objection were offered. No objection being offered, the President declared the minutes approved.

The Secretary announced that since our last meeting the following additions to membership of the Institute had occurred: five Family Memberships; Nineteen Resident Memberships; Two Non-Resident Memberships and one Student Membership; he also announced with great regret the death of Fifteen Members.

The Secretary stated that at the meeting of the Board of Managers held on October eighteenth, Mr. Wilfred W. Fry of Philadelphia had been unanimously elected to membership on the Board of Managers to succeed the late Mr. Cyrus H. K. Curtis.

The Secretary then stated that the official opening of the Fels Planetarium would be held on the afternoon of Wednesday, November first, at three-thirty o'clock and that beginning on Monday, November sixth regular public sessions of the Planetarium would be held. As arranged at present, the schedule of these exercises follows:

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| Monday<br>Tuesday<br>Wednesday<br>Thursday<br>Friday | 11:00 A.M.<br>3:30 P.M.<br>8:00 P.M. |
| Saturday | 11:00 A.M.<br>2:30 P.M.<br>4:00 P.M.<br>8:00 P.M. |
| Sunday | 2:30 P.M.<br>4:00 P.M.<br>8:00 P.M. |

He further made the statement that various sections of the Museum were in such a state of advancement that it was now expected to open them to the public on December fifth. Other announcements concerning these openings will be made at a later time.

There being no further business, the President introduced the speaker of the evening, Dr. V. K. Zworykin of the Engineering Department, Research Division, RCA Victor Company, Camden, New Jersey, who spoke on the subject "Television." Dr. Zworykin spoke in an absorbingly interesting way concerning the new system of television which has been developed by him and his associates in the RCA Victor Corporation.

The talk was followed by an interesting discussion by the various members of the audience and by an expression of appreciation by the President.

The meeting adjourned at nine thirty-nine P.M.

HOWARD MCCLENEHAN,
*Secretary.*

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## COMMITTEE ON SCIENCE AND THE ARTS.

(*Abstract of Proceedings of Stated Meeting held Wednesday, October 11, 1933.*)

HALL OF THE COMMITTEE,
PHILADELPHIA, OCTOBER 11, 1933.

MR. THEOBALD F. CLARK *in the Chair.*

The following reports were presented for first reading:

No. 2881: Mercury Compressor Evolved from Archimedes Screw Pump.

No. 2968: Machines and Apparatus for Generating Double Helical Gears.

GEO. A. HOADLEY,
*Secretary to Committee.*

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## MEMBERSHIP NOTES.

### ELECTIONS TO MEMBERSHIP.

(*Stated Meeting, Board of Managers, October 18, 1933.*)

FAMILY MEMBERSHIP.

MR. WALTON FORSTALL, 1800 North Ninth Street, Philadelphia, Pa.

MR. FREDERIC S. WHITTAKER, Meadowbrook Lane, W. Stenton Ave., Chestnut Hill, Philadelphia, Pa.

RESIDENT.

PROFESSOR CHARLES A. BAREUTHER, Teacher, Central High School, Philadelphia, Pa. For mailing: 5533 Greenway Avenue, Philadelphia, Pa.

MR. GREVILLE BATHE, Mechanical Engineer, Philadelphia, Pa. For mailing: N.E. cor. 49th & Regent Sts., Philadelphia, Pa.

PROFESSOR GEORGE EMERSON CROFOOT, Educator, University of Pennsylvania, For mailing: Engineering Building, University of Pennsylvania, Philadelphia, Pa.

MR. GEORGE F. DOWNING, Teaching, LaSalle College, For mailing: La Salle College, Logan Station, Philadelphia, Pa.

MR. GEORGE S. GARDNER, Chemist, Brewer & Gardner, For mailing: 5920 No. Third Street, Philadelphia, Pa.

DR. WALTER H. HARTING, Chemist, Sharp & Dohme, Inc., Broad and Wallace Streets, Philadelphia, Pa.

MR. HARRY L. HAWKINS, Advertising, The Evening Bulletin, Philadelphia, Pa. For mailing: 316 Kent Road, Cynwyd, Pennsylvania.

MR. KARL S. HOWARD, Mechanical Superintendent, General Steel Castings Corp., Eddystone, Pa. For mailing: Merion Cricket Club, Haverford, Pa.

MR. RALPH G. PUTNAM, Merchant, J. E. Caldwell and Company, Philadelphia, Pa. For mailing: 7201 Wayne Avenue, Germantown, Pa.

MR. JOSEPH W. SIMPSON, Newspaper, The Evening Bulletin, Philadelphia, Pa.

DR. FLOYD T. TYSON, Chemistry, Temple University, Philadelphia, Pa. For mailing: 111 E. Browning Road, Collingswood, N. J.

NON-RESIDENT.

MR. J. M. FARNUM, Chemist, Taylor & Company. Inc., Norristown, Pa.

CHANGES OF ADDRESS.

DR. MILDRED ALLEN, 14 Silver Street, South Hadley, Mass.

MR. G. R. HALL, Denville, P. O., Denville, Morris County, N. J.

MR. WALTER G. HENSEL, University Club, 546 Delaware Avenue, Buffalo, New York.

MR. FREDERIC V. HETZEL, 103 Dean Street, West Chester, Pa.

MR. WILLIAM HURST, Front and Jackson Streets, Media, Pa.

MR. ROLAND W. JOHNSON, 418 East Phil-Ellena Street, Germantown, Philadelphia, Pa.

MR. J. L. KILPATRICK, 1165 Park Avenue, New York, New York.

MR. WILLIAM HOWARD MCCOY, 16522 LaSalle Boulevard, Detroit, Michigan.

MR. ERNEST R. RECHEL, 7339 Whitaker Avenue, Philadelphia, Pa.

REAR ADMIRAL ELLIOT SNOW, C. C., U. S. N., Apt. B-2, Bryn Mawr Court, Bryn Mawr, Pa.

MR. GEORGE B. STANTON, 310 Windsor Place, Brooklyn, New York.

MR. CLIFFORD J. SUMMERS, 446 Diversey Parkway, Chicago, Illinois

## NECROLOGY.

**Mr. John Bancroft,** Wilmington, Del.
**Dr. George K. Burgess,** Washington, D. C.
**Dr. A. M. Comey,** Cambridge, Mass.
**Mr. Arthur Falkenau,** Stamford, Conn.
**Mr. H. J. Kaltenthaler,** Overbrook, Philadelphia, Pa.
**Mr. Spencer Otis,** Chicago, Illinois.
**Mr. J. M. Spitzglass,** Chicago, Illinois.

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## LIBRARY NOTES.

### RECENT ADDITIONS.

American Institute of Chemical Engineers. Twenty-five Years of Chemical Engineering Progress. Silver Anniversary Volume. 1933.

BARNARD, WILLIAM N., FRANK O. ELLENWOOD, AND CLARENCE F. HIRSHFELD. Elements of Heat-Power Engineering. Part II: Steam Generating Apparatus and Prime Movers, Fuels, Combustion, and Heat Transmission. Third edition. 1933.

BURNHAM, T. H. Special Steels. A Concise Treatise on the Constitution, Manufacture, Working, Heat Treatment and Applications of Alloy Steels. For Students, Operators, and Users of Special Steels. Chiefly founded on the researches regarding alloy steels of Sir Robert Hadfield. Second edition. 1933.

DAVID, PIERRE. Les Radio-Communications Modernes. 1932.

DOHSE, H., UND H. MARK. Die Adsorption von Gasen und Dämpfen an festen Körpern. 1933. Hand- und Jahrbuch der chemischen Physik. Band 3, 1 Teil, Abschnitt 1.

EKKERT, LAD. Erkennung organischer Verbindungen im besonderen von Arzneimitteln. 1933. Die chemische Analyse XXXII Band.

FRANCK, SIEGFRIED. Messentladungsstrecken. (Ionenstrecken.) 1931.

FREUDENBERG, K. Tannin, Cellulose, Lignin. Zugleich zweite Auflage der "Chemie der natürliche Gerbstoffe." 1933.

GEIGER, H., UND KARL SCHEEL. Handbuch der Physik. Band XXIII, zweiter Teil: Röntgenstrahlung, ausschliesslich Röntgenoptik. 1933.

GNAMM, H. Die Gerbstoffe und Gerbmittel. Zweite Auflage, 1933. Chemie in Einzeldarstellungen XII Band.

GRAY, ALEXANDER. Principles and Practice of Electrical Engineering. Revised by G. A. Wallace. Fourth edition. 1933.

GRIMSEHL, E. A Textbook of Physics. Edited by R. Tomaschek. Authorized translation from the seventh German edition by L. A. Woodward. Vol. II. Heat and Sound. 1933.

HAYNES, WILLIAMS. Chemical Economics. 1933.

HEVESY, GEORG V., UND ERNST ALEXANDER. Praktikum der chemischen Analyse mit Röntgenstrahlen. 1933.

HOCKER, EDWARD W. Germantown, 1683–1933. The Record That a Pennsylvania Community Has Achieved in the Course of 250 Years: Being a History of the People of Germantown, Mount Airy and Chestnut Hill. 1933.

KRÖGER, MARTIN. Grenzflächen-Katalyse. 1933.

LINDSAY, ROBERT BRUCE. Physical Mechanics. An Intermediate Text for Students of the Physical Sciences. 1933.

LÖWE, FRITZ. Optische Messungen des Chemikers und des Mediziners. Zweite erweiterte und neubearbeitete Auflage. 1933. Technische Fortschrittsberichte, Bd. VI.

LUBBOCK, CONSTANCE A. The Herschel Chronicle: the Life Story of William Herschel and his Sister, Caroline Herschel. 1933.

MAXTED, EDWARD B. Catalysis and its Industrial Applications. 1933.

MORECROFT, JOHN H. Electron Tubes and their Application. 1933.

MOYER, JAMES A., AND RAYMOND U. FITTZ. Air Conditioning. First edition. 1933.

National Amateur Press Association. Year-Book, 1886–1887.

NEMÉNYI, PAUL. Wasserbauliche Strömungslehre. 1933.

Organic Syntheses: an Annual Publication of Satisfactory Methods for the

Preparation of Organic Chemicals. Volume 13, W. H. Carothers, Editor-in-Chief. 1933.

SÄNGER, EUGEN. Raketen-Flugtechnik. 1933.

SMITH, MORTON MOTT-. Heat and its Workings. 1933.

STOHN, RUDOLF. Temperaturregler. 1933.

STREETER, ROBERT L., AND LESTER C. LICHTY. Internal-Combustion Engines: Theory, Analysis and Design. A Treatise on Internal-Combustion Engines for Engineers and Students in Engineering. Fourth edition. 1933.

WAGNER, C. F., AND R. D. EVANS. Symmetrical Components as Applied to the Analysis of Unbalanced Electrical Circuits. First edition. 1933.

WEISER, HARRY BOYER. Inorganic Colloid Chemistry. Volume I: the Colloidal Elements. 1933.

WIZINGER, R. Organische Farbstoffe. Anleitung zum schrittweisen Eindringer in die Farbenchemie auf koordinationstheoretischer Grundlage. 1933.

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## BOOK REVIEWS.

LES RADIO-COMMUNICATIONS MODERNES, par Pierre David, Docteur Es-sciences, ingenieur en chef au laboratoire national de radio-electricite. 149 pages, illustrations, 14 × 21 cms. Paris, Librairie J.-B. Bailliere et Fils, 1932. Price 20 fr.

Following the genius of the French this book is printed on paper that does not involve a fortune and is bound with a paper back, in this way making the matter particularly accessible to those who have interest in radio communications. Today this includes (hasty calculation) 7/8 of the civilized world.

The casual school-boy who expects easy browsing will find this book refreshing his French—scientific French is not very difficult—and also illuminating the intricacies of wireless communication. If he does not know a word of French and not too many words of his mother tongue he will find the many illustrations of all kinds of apparatus useful to the T. S. F. (Telegraphie sans fil—telegraphy without wire—alias wireless).

A country that is favored with such technical writings is destined to be informed in this changing world. The poorly printed cuts stumble anyone but there is always something inspiriting to me to have to cut the edges of the page. It is signal for fresh attack into a new trench of learning.

H. W. ELKINTON.

HEAT AND ITS WORKINGS, by Morton Mott-Smith, Ph.D., 239 pages, illustrations, 13.5 × 19.5 cms. New York, London, D. Appleton & Company, 1933. Price $2.00.

Those who have read "This Mechanical World" will find the same happy treatment in another field of physical forces. The workings of heat and obversely of cold are at us constantly. With complete disregard for the effects of atmospheric pressure, moisture and temperature, none would be long for this world. Mankind accommodates these things grudgingly. Occasionally a specimen of homo sapiens desires to know more about the forces that provoke so much temper

and so very much conversation. "Heat and Its Workings" will help such a person.

Dr. Mott-Smith has not attempted, however, to explore primarily climate, rather the "temperature sense, the expansion of bodies," gas, calories and latent heat as in chapters I to V. He also uses many good illustrations. Amongst other well selected cuts there is the time honored expose of Hero's Temple Trick. The book abounds in ready allusions to the work of others in the near and deep past, able diagrams of water and ice and gas and steam, complicated drawings that show depth as well as plane values. One of the most intriguing is Fig. 46, that shows water boiling in a paper box over a flame. Of course no book on physics can escape without plungers and "U" tubes compressing or expanding something.

The book is well worth reading, well bound and readably printed. It adds to an intelligent understanding of the Workings of Heat in the language which anyone can follow.

H. W. ELKINTON.

SIR PRAFULLA CHANDRA RAY, Seventieth Birth Day Commemoration Volume, Journal of the Indian Chemical Society, Special Number. 362 pages, illustrations, 17 × 26 cms. Calcutta, University Press, 1933.

The reviewer believes it will be some time before he again sees a special number of a chemical society journal similar to this. The Indian Chemical Society may well be proud of its Commemoration Volume dedicated to a world-renowned pioneer of chemical research in India, Sir Prafulla Chandra Ray. This patriotic chemist is not only noted for his labors in pure and applied chemistry but also for his work on the history of Hindu chemistry. All this and much more is said in the dedicatory prologue extolling the brilliance, the integrity, the philanthropy and humanism of a beloved and revered septuagenarian chemist.

To get back to the periodical itself, it is very, very seldom that such a collection of authors will be found between the covers of a chemical journal. There are articles by G. T. Morgan, T. S. Wheeler, F. G. Donnan, James W. McBain, H. Freundlich, Friedrich Emich, Hans Pringsheim, Franz Fisher, J. C. Ghosh, N. R. Dhar and many other Indian chemists well known in their own country but whose names may not sound so familiar to their brother chemists in distant America. After reading the list of authors all chemists will have prepared themselves for a series of papers touching upon almost every phase of chemical science and dealing with the subjects in a clear, concise and authoritative manner.

In all, there are thirty-six papers of which only brief mention can be made here. Colloid chemistry has always received considerable attention from Indian chemists and is well represented. Articles upon Dialysis, Coagulation, Cataphoresis, Coarse Particle Systems, Inorganic Jellies, Gelation, Mechanism of Changes in Colloidal States are to be found. In the realm of inorganic and physical chemistry there are treatises upon such subjects as: Thermodynamic Functions of Radiation, Constitution of Molecular Compounds, Temperature and Diamagnetism, Optical Isomers, Adsorption, Gaseous Explosions, Photochemical Radiations, Optical Studies on Uranium Compounds, Cryolysis in Biological Problems and Stereochemistry. The biological sciences are recognized through articles on: Nitrification in Soil and in Atmosphere, Proteolytic Enzymes in

Plants, Chemotherapy of Organo-metalloid Compounds, Mineral Metabolism of Farm Animals and Observations on the Chemistry of Vitamins $B_1$ and $B_2$. Other papers more purely organic in nature deal with the: Dehydrogenation of Pyridine, Pyrolytic Changes of Ethane, Synthesis of Chromones, Utilization of Methane, Detection of some Polyhydroxyphenols, Interaction of Thiocarbamides with Bromine, Keto-methylene Condensation, Synthesis of Anthraquinones in Nature, Latest Developments on Polysaccharides and Relation of Carbon to Various Gases. A paper that escapes classification but withal very interesting and rather unique was submitted by Friedrich Emich: Einige Bemerkungen über die quantitative Ermittlung sehr kleiner Stoffmengen.

T. K. CLEVELAND.

THE COMMUTATOR MOTOR, by F. J. Teago, D.Sc., M.I.E.E., Robert Rankin Professor of Electrical Machinery in the University of Liverpool, Chartered Electrical Engineer. 80 pages, illustrations, 11 × 17.5 cms. New York, E. P. Dutton & Co., Inc. Price $1.10.

This handy pocket-size volume is a member of a series of "Monographs on Physical Subjects." Induction motors fitted with commutators are still a relatively recent development and the author deserves a vote of gratitude for having brought together much essential information hitherto available only in widely scattered writings.

The object of this monograph is not to provide a detailed mathematical exposition on the commutator-induction motor but rather an elucidation of the principles of operation and a critical appraisal of the several structural modifications. It is essential that the reader be familiar with the circle diagram of the ordinary induction motor since these newer developments represent merely the addition of a commutator and possibly an extra winding. For those not familiar with the circle diagram, the author explains its derivation in the appendix.

The chapters represent the various phases of the subject involved. The first three dealing with the idea of speed control, rotating magnetic fields and the power factor idea are purposely introductory in nature. One of the main purposes of the commutator is to bring about a power factor correction. How this is done forms the subject of the chapter on the Mechanism of Energy Transference. Certain types of commercial motors are provided with a commutator to permit operation at variable speeds while others run at constant speeds and utilize the commutator to correct the power factor. Still others are fitted with what is known as the Schrage mechanism which makes possible a combined speed and power factor control. A very interesting chapter on Cascade Operation deals with the benefits made possible through hooking up a commutatored induction motor with an ordinary motor. The value of the commutator motor is also pointed out in instances of generator operation and where periodic heavy loads require an induction motor to operate with increasing amounts of slip.

T. K. CLEVELAND.

THE METHOD OF DIMENSIONS, by Alfred W. Porter, D.Sc., F.R.S., Emeritus Professor of Physics in the University of London. 79 pages, illustrations, 11 × 17.5 cms. New York, E. P. Dutton & Co., Inc. Price $.85.

A member of Methuen's Monographs on Physical Subjects, this little volume represents, so far as the author is aware, the first treatise on the "Method of

Dimensions" in his native country. The prime object of the book is to show how the many equations describing physical facts may be expressed in terms of the more fundamental or absolute dimensions. The reader can appreciate the significance of accomplishing such an object. The establishment of a fundamental equation covering all physical phenomena is ever a paramount goal for physicists.

The subject of "Fluid Flow" is discussed in relation to flow in pipes, motion of solid bodies in viscous fluids, flow past obstacles and flow through apertures. Surface tension manifestations are recognized by the "rise in a capillary tube" and by "falling drops." Vibrating systems embrace tuning forks and the like, planetary motion, ripples and waves. Temperature and heat effects call for recognition of heat interchange, also the relations between temperature and viscosity. In Electricity and Magnetism the close relationship between equations of electrostatic, magnetic and electromagnetic forces are demonstrated. In conclusion the author mentions the probability of additional future successes in reducing several of the more complicated physical relations to expressions containing only the fundamental quantities: mass, length and time.

T. K. CLEVELAND.

ALLEN'S COMMERICAL ORGANIC ANALYSIS, A TREATISE ON THE PROPERTIES, MODES OF ANALYSIS, AND PROXIMATE ANALYTICAL EXAMINATION OF THE VARIOUS ORGANIC CHEMICALS AND PRODUCTS, VOLUME X, HAEMOGLOBIN AND ITS DERIVATIVES, ALBUMINOIDS OR SCLEROPROTEINS. . . . Editor, C. Ainsworth Mitchell. Fifth edition, revised and partly rewritten, 817 pages, illustrations, tables, 15.5 × 24 cms. Philadelphia, P. Blakiston's Son & Co., Inc., 1933. Price $7.50.

It is finished: The tenth and final volume of the fifth edition of "Allen's Commercial Organic Analysis" is off the press. After looking over the list of contents for the preceding nine volumes, one marvels that sufficient knowledge for another yet remained. Back in volume eight the "proteins" were introduced; nor was the subject exhausted when volume nine had attained its page quota. So, volume ten leads off with "Hæmoglobin and its Derivatives." The chemistry of these compounds is discussed with considerable detail and is followed with an extremely comprehensive section dealing with the Medico-legal Examination of Blood.

Under the heading "Albuminoid or Scleroproteins" will be found dissertations on such organic products as elastin (from tendon), collagen (from bone and hide), keratin (from hoof and horn). The collagens include Gelatin, Isinglass from swimming bladders of fish and Sericin, the gummy protecting sheaf of silk. Elastin, found in animal ligaments, is classified along with Fibroin, constituting silk and spider's webs. Still a third class embraces Chitin, the chief constituent of invertebrata coverings such as the lobster and crab; also Spongin from sponges. Considerable space is accorded such commercially important derivatives as glue, and gelatin. Silk, both natural and artificial, is dealt with extensively, particularly from the standpoint of type identification. The "Keratins" include methods for examining and identifying furs, hair and wool.

A very timely section is "The Examination of Foodstuffs for Vitamins." The knowledge of vitamins grows so rapidly that it is impossible for a work of this

type to keep abreast with developments. However, a number of standard diets have been established for the purpose of determining sources of vitamins and estimating their abundance. An appendix contains the full report of the Health Organization of the League of Nations on the preparation of International Vitamin Standards.

Hormones, the active principles of the glands of internal secretion, form the centers of much present-day research activity. The constitution of thyroxine and adrenaline have been established beyond a doubt. Nevertheless, the quantitative estimation of these and other hormones generally is done physiologically. Insulin and the pituitary hormones hold a commanding position and sufficient is now known about the hormones of the sex glands to warrant their recognition.

"The Identification of Unknown Woods and Charcoals" probably forms the most unique section of the whole edition. The methods of identification are described at length: anatomical, physical and chemical. The author of this section is English and better acquainted with European woods. It is to be hoped that by the time the sixth edition is ready there will be a supplement on American woods.

"Pectic Substances" have attained considerable commercial recognition in the past several years. Pectin is held responsible for gel formation in fruit juices and its extraction with subsequent use in pectin-sugar fruit jellies is an important topic.

Volume Ten is extremely valuable for another reason. It contains a General Reference Index of the whole ten volumes. Space does not permit the inclusion of an Authors' Index, yet many names associated with special processes will be found in the General Index.

T. K. CLEVELAND

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS.

Report No. 459, The N. A. C. A. Full-Scale Wind Tunnel, by Smith J. DeFrance, 10 pages, illustrations, 23 × 29 cms. Washington, Superintendent of Documents, 1933. Price five cents.

This report gives a complete description of the full-scale wind tunnel of the National Advisory Committee for Aeronautics. The tunnel is of the double-return flow type with a 30 by 60 foot open jet at the test section. The air is circulated by two propellers 35 feet 5 inches in diameter, located side by side, and each directly connected to a 4,000-horsepower slip-ring induction motor. The motor control equipment permits varying the speed in 24 steps between 25 and 118 miles per hour. The tunnel is equipped with a 6-component balance for obtaining the forces in 3 directions and the moments about the 3 axes of an airplane. All 7 dial scales of the balance system are of the recording type, which permits simultaneous records to be made of all forces.

The tunnel has been calibrated and surveys have shown that the dynamic-pressure distribution over that portion of the jet which would be occupied by an airplane having a wing span of 45 feet is within $\pm 1\frac{1}{2}$ per cent. of a mean value. Based on the mean velocity of 118 miles per hour at the jet, the ratio of the kinetic energy per second to the energy input to the propellers per second is 2.84. Since it is generally recognized that a long open jet is a source of energy loss, the above figure is considered very satisfactory.

Comparative tests on several airplanes have given results which are in good agreement with those obtained on the same airplanes in flight. This fact, together with information obtained in the tunnel on Clark Y airfoils, indicates that the flow in the tunnel is satisfactory and that the air stream has a very small amount of turbulence.

Report No. 462, Tests of Nacelle-Propeller Combinations in Various Positions with Reference to Wings. III. Clark Y Wing—Various Radial-Engine Cowlings—Tractor Propeller, by Donald H. Wood, 42 pages, tables, illustrations, 23 × 29 cms. Washington, Superintendent of Documents, 1933. Price ten cents.

This report is the third of a series giving the results obtained in the 20-foot wind tunnel of the National Advisory Committee for Aeronautics on the interference drag and propulsive efficiency of nacelle-propeller-wing combinations. The first report gave the results of the tests of an N. A. C. A. cowled air-cooled engine nacelle with tractor propeller located in 21 positions with reference to a thick wing. The second report gave the results for several engine cowlings and nacelles with tractor propeller located in four positions with reference to the same wing. The present report gives results of tests of the same nacelles and cowlings in the same positions with reference to a smaller wing of Clark Y section.

The results of the tests with the Clark Y wing are in general agreement with those obtained using a thick wing. The N. A. C. A. cowled nacelle located directly ahead of the wing is the best tractor-nacelle arrangement. Analysis of the results shows that the net efficiency is but little affected by the airfoil section of the wing if the nacelles are located the same fraction of the chord from the leading edge. The gain in efficiency due to cowling the engine is so much greater than the gain due to proper nacelle location that it is advisable to cowl radial engines carefully before attempting to take advantage of the favorable effects of locating the nacelle ahead of the wing. The proper location of nacelles and careful cowling are important in the high-speed range of flight, but in the lower speed ranges there is little advantage of one nacelle position or cowling over another.

Report No. 464, Negative Thrust and Torque Characteristics of an Adjustable-Pitch Metal Propeller, by Edwin P. Hartman, 13 pages, illustrations, 23 × 29 cms. Washington, Superintendent of Documents, 1933. Price five cents.

This paper presents the results of a series of negative thrust and torque measurements made with a 4-foot-diameter model of a conventional aluminum-alloy propeller. The tests were made in the 20-foot propeller-research tunnel of the National Advisory Committee for Aeronautics.

The propeller was tested for thrust and torque through a blade-angle range from 22° to −23° at 0.75 radius and a $V/nD$ range from zero to infinity, while mounted in front of a cowled radial-engine nacelle. With this arrangement the drag of the propeller was also measured through a blade-angle range from 0° to 90° while locked in a vertical position. Additional tests were made with the propeller and nacelle located in two positions with respect to both a monoplane wing and a biplane cellule, in which smaller ranges of blade angles and values of $V/nD$ were covered.

The results show that the negative thrust is considerably affected by the shape and size of the body behind the propeller, that the maximum negative thrust increases with decrease in blade-angle setting, and that the drag of a locked propeller may be greatly reduced by feathering it into the wind. Several examples of possible applications of the data are given.

Report No. 465, Determination of the Theoretical Pressure Distribution for Twenty Airfoils, by I. E. Garrick, 15 pages, illustrations, 23 × 29 cms. Washington, Superintendent of Documents, 1933. Price five cents.

This report gives the theoretical distribution of pressure at lift coefficients of 0, 0.5 1.0, and 1.5 for 20 airfoils, calculated on the basis of a rigorous potential theory of arbitrary airfoils. It also provides tables from which the characteristics of the airfoils for any angle of attack in 2-dimensional potential flow are readily calculable. The theoretical values of the angles of zero lift, the lift and moment coefficients, and the ideal angles of attack are listed and some comparisons with experiment are indicated. Some of the well-known characteristics and properties of airfoils are accounted for in terms of the theoretical pressure-distribution curves. Qualitative deductions are made concerning the causes of breakdown of potential flow and the efficiency of the airfoil in viscous flow. The results presented may be of value in predicting structural loads and also in a correlation of theoretical pressure gradients with profile resistance.

Report No. 468, The Interference Between Struts in Various Combinations, by David Biermann and William H. Herrnstein, Jr., 12 pages, illustrations, 23 × 29 cms. Washington, Superintendent of Documents, 1933, Price five cents.

This report presents the results of tests made in the N. A. C. A. 7– by 10-foot wind tunnel to determine the interference drag arising from various arrangements of streamline struts and round struts, or cylinders. Determinations were made of the interference drag of struts spaced side by side, struts in tandem, tandem struts encased in a single fairing, a strut intersecting a plane, and struts intersecting to form a V. Three sizes of struts were used for most of the tests.

These tests show that the interference drag arising from struts in close proximity may be of considerable magnitude, in some instances amounting to more than the drag of the struts themselves.

Report No. 469, Increasing the Air Charge and Scavenging the Clearance Volume of a Compression-Ignition Engine, by J. A. Spanogle, C. W. Hicks, and H. H. Foster, 12 pages, illustrations, 23 × 29 cms. Washington, Superintendent of Documents, 1933. Price five cents.

The object of the investigation presented in this report was to determine the effects of increasing the air charge and scavenging the clearance volume of a 4-stroke-cycle compression-ignition engine having a vertical disk form of combustion chamber.

Boosting the inlet-air pressure with normal valve timing increased the indicated engine power in proportion to the additional air inducted and resulted in smoother engine operation with less combustion shock.

Scavenging the clearance volume by using a valve overlap of 145° and an inlet-air boost pressure of approximately 2½ inches of mercury produced a net

increase in performance for clear exhaust operation of 33 per cent. over that obtained with normal valve timing and the same boost pressure. The engine tests indicate that, with a large valve overlap, 2½ inches of mercury boost pressure is sufficient to scavenge completely the clearance volume, and that the increase in engine power effected by scavenging the clearance volume for a constant fuel quantity, is more than twice what could be obtained from the additional air charge alone. The improved combustion characteristics result in lower specific fuel consumption, and a clearer exhaust. The starting and idling characteristics were not affected by variation in boost pressure or by the use of scavenging with a large valve overlap.

Analysis of the exhaust of several compression-ignition engines showed the carbon monoxide gas content to be less than one half of 1 per cent. when operating with a clear exhaust.

Report No. 471, Performance of a Fuel-Injection Spark-Ignition Engine Using a Hydrogenated Safety Fuel, by Oscar W. Schey and Alfred W. Young, 10 pages, illustrations, 23 × 29 cms. Washington, Superintendent of Documents, 1933. Price five cents.

This report presents the performance of a single-cylinder test engine using a hydrogenated safety fuel. The safety fuel has a flash point of 125° F. (Cleveland open-cup method), which is high enough to remove most of the fire hazard, and an octane number of 95, which permits higher compression ratios to be used than are permissible with most undoped gasolines. The fuel was injected into the engine cylinder, except for a few comparative runs with gasoline, when a carburetor was used. The tests were made with compression ratios of 5.85 and 7.0, valve timings giving 30° and 130° overlap, inlet pressures from atmospheric to 6 inches of mercury boost, and engine speeds from 1,250 to 2,200 r.p.m. Under similar conditions the power obtained with the safety fuel was the same as that obtained with gasoline, whereas the fuel consumption was from 5 to 10 per cent. higher. With a compression ratio of 7.0, a valve overlap of 130 crankshaft degrees, and a boost pressure of 2 inches of mercury, the safety fuel gave a brake mean effective pressure of 175 pounds per square inch with a fuel consumption of 0.50 pound per brake horsepower hour.

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## PUBLICATIONS RECEIVED.

*Rayleigh's Principle and its Applications to Engineering*, by G. Temple and W. G. Bickley, 156 pages, tables, diagrams, 14.5 × 22 cms. New York, Oxford University Press, London, Humphrey Milford, 1933. Price $4.50.

*The Principles of Metallurgy*, by Donald M. Liddell and Gilbert E. Doan, 626 pages, tables, illustrations, 15.5 × 23.5 cms. New York and London, McGraw-Hill Book Company, Inc., 1933. Price $5.50.

*Industrial Heat Transfer*, by Alfred Schack, translated from the German by Hans Goldschmidt and Everett P. Partridge, 371 pages, diagrams, tables, 15 × 24 cms. New York, John Wiley & Sons, Inc., London, Chapman & Hall, Ltd., 1933. Price $5.00.

*Wireless Receivers, the Principles of Their Design*, by C. W. Oatley, with a

preface by O. W. Richardson, 103 pages, diagrams, 11 × 17.5 cms. London, Methuen & Co., Ltd., New York, E. P. Dutton & Co. Inc., Price $.85.

*The Romance of Research*, by L. V. Redman and A. V. H. Mory, 149 pages, 13 × 19 cms. Baltimore, The Williams and Wilkins Company, 1933. Price $1.00.

*The Elements of the New Quantum Mechanics*, by Otto Halpern and Hans Thirring, translated from the German by Henry L. Brose, 215 pages, diagrams, 14.5 × 23 cms. London, Methuen & Co., Ltd., New York, E. P. Dutton & Co., Inc. Price $3.25.

*Modern Thermodynamics by the Methods of Willard Gibbs*, by E. A. Guggenheim, with a preface by F. G. Donnan, 206 pages, diagrams, 14.5 × 23 cms. London, Methuen & Co., Ltd., New York, E. P. Dutton & Co., Inc.

*The Voice, Its Production and Reproduction*, by Douglas Stanley, and J. P. Maxfield, 287 pages, illustrations, 14.5 × 22.5 cms. New York, Pitman Publishing Corp., Price $2.50.

*U. S. Bureau of Mines, Minerals Yearbook*, 1932–33, 819 pages, tables, 15 × 23.5 cms. Washington, Government Printing Office, 1933. Price $1.25. Mineral Resources of the United States, 1930, Part I—Metals, 1142 pages, tables, 15.5 × 23.5 cms. Washington, Government Printing Office, 1933. Price $1.50. Mine Report, Gold, Silver, Copper, Lead and Zinc in Utah in 1931, by C. N. Gerry and Paul Luff, 31 pages, tables, 15 × 23 cms. Washington, Government Printing Office, 1933. Price five cents. Economic Paper 16, Potash, by Bertrand L. Johnson, 78 pages, tables, 15 × 23 cms. Washington, Government Printing Office, 1933. Price ten cents.

*Royal Institution of Great Britain, Weekly Evening Meeting:* May 19, 1933, Planet Studies at the Lowell Observatory, by V. M. Slipher, 18 pages, plates, 14 × 21.5 cms. London, Institution, 1933.

*National Advisory Committee for Aeronautics*, Technical Notes, No. 470, A Complete Tank Test of a Model of a Flying-Boat Hull—N. A. C. A. Model No. 11-A, by John B. Parkinson, 9 pages, tables, illustrations, 20 × 26.5 cms. Washington, Committee, 1933. No. 471, A Complete Tank Test of a Model of a Flying-Boat Hull—N. A. C. A. Model 16, by James M. Shoemaker, 8 pages, tables, diagrams, 20 × 26.5 cms. Washington, Committee, 1933. No. 472, The Effect of Partial-Span Split Flaps on the Aerodynamic Characteristics of a Clark Y Wing, by Carl J. Wenzinger, 5 pages, tables, illustrations, 20 × 26.5 cms. Washington. Committee, 1933.

## CURRENT TOPICS.

**Chloride and Sulphate Content of Rain Water.**—W. D. COLLINS AND K. T. WILLIAMS (*Ind. and Eng. Chem.*, Ind. Ed., 1933, XXV, 944–945) conclude that the greater part of the water which reaches the surface of the United States as rain or snow contains at that time less than 0.3 part of chloride anion per million, and not more than 2 parts of sulphate anion per million. These concentrations correspond, respectively, to 0.07 pound of chloride anion and 0.15 pound of sulphur per acre per inch of rain-fall.

J. S. H.

**Vitamin A Content of Foods and Feeds.**—Vitamin fat-soluble A protects the animal from infections and from the eye disease known as xerophthalmia. It is closely related to the yellow pigment carotene. The units of vitamin A are determined biologically, 1 unit is the daily dose sufficient to cause an immature albino rat, which has been depleted of vitamin A, to gain 24 grams in body weight in a period of 8 weeks when on a standard ration. Xerophthalmia must not develop during this period. G. S. FRAPS AND RAY TREICHLER (Texas Agric. Exp. Sta. Bull. No. 477, 34 pages, 1933) have assayed over 100 foods and feeds for vitamin A. A decrease in vitamin A content occurred during the drying of moist foods, and during the storage of alfalfa, dried whole milk, yellow corn and other foods. A direct quantitative relationship existed between the yellow pigmentation of corn and its vitamin A content. The content of this vitamin in butter and eggs depended upon its content in the rations of the cows and chickens which produced them. The cheapest sources of vitamin A for human food were bananas, carrots, yellow corn, cod liver oil, collard greens, liver, mustard greens, spinach, yellow sweet potatoes, and turnip greens; they yielded 1000 vitamin units at a cost between 0.1 and 0.5 cent. Canned spinach, canned carrots, and canned mustard were slightly more expensive, yet served as relatively low-priced sources of the vitamin. Butter, eggs, cheese, milk, and dried green peas were relatively economical sources, while asparagus, cabbage, head lettuce, and canned peaches were expensive sources.

The daily vitamin A requirement of animals per pound of body weight was found to be as follows: Growing rats, for maintenance 4 units, for growth and maintenance 6 units; white Leghorn pullets,

for maintenance 32 units. For each vitamin A unit in the egg, the hen required 6.3 extra units in her ration. Every human being required at least 1000 units, possibly additional units for higher vigor and better health; the 1000 units were available at a low price in comparatively small quantities of collard greens, turnip greens, mustard greens, or spinach, or in somewhat larger amounts of bananas, carrots, yellow corn, or sweet potatoes. The ration apparently must include green pasture plants in order to produce milk and eggs rich in vitamin A. Cottonseed meal, hays, fodders, and sorghum silage were low in vitamin A content.

J. S. H.

**Atomic Weight of Potassium.**—GREGORY PAUL BAXTER AND WILLIAM MARSHALL MACNEVIN (*Jour. Am. Chem. Soc.*, 1933, LV, 3185–3190) have determined the atomic weight of potassium by use of the ratio of potassium chloride to silver. They conclude, from a series of 33 determinations, that the atomic weight of potassium has a value very close to 39.096.

GREGORY PAUL BAXTER AND CHESTER M. ALTER (*Jour. Am. Chem. Soc.*, 1933, LV, 3270–3271), have also used the ratio of potassium chloride to silver to determine the atomic weight of both common potassium and the less volatile or heavy potassium of Hevesy. The common potassium yielded a value of 39.096, the heavy potassium a value of 39.109. These results indicate the presence of 7.3 per cent. of the isotope $K_{41}$ in the heavy potassium.

J. S. H.

**Solubility of Lead Sulphate.**—R. B. PURDUM AND H. A. RUTHERFORD, JR. (*Jour. Am. Chem. Soc.*, 1933, LV, 3221–3223) have determined the solubility of lead sulphate in water, using large volumes (6 liters) of solvent, and four different samples of the salt. The solubility was 43.5 ± 0.1 mg. of lead sulphate per liter of water.

J. S. H.

**Biological Activity of Isotopes.**—GILBERT N. LEWIS (*Jour. Am. Chem. Soc.*, 1933, LV, 3503–3504) describes the influence of the hydrogen isotope in water on the sprouting of tobacco seed. Seeds readily germinated in ordinary water $H_2O$, but failed to develop in "heavy" water $H_2H_2O$. Seeds were placed in water in which one-half of the hydrogen was $H_2$; this water had the molecular formula $H_1H_2O$; these seeds grew, but only one-half as rapidly, as seeds placed in ordinary water.

J. S. H.

**New Synthesis of Camphor.**—JOHN J. RITTER (*Jour. Am. Chem. Soc.*, 1933, LV, 3322–3326) has achieved a new synthesis of camphor. Bornyl chloride is converted into isobornylaniline, then, by means of a nickel catalyst, into camphor anil. This is then hydrolyzed to obtain camphor.

J. S. H.

**Prolactin: a New Hormone.**—OSCAR RIDDLE, ROBERT W. BATES, AND SIMON W. DYKSHORN (*Am. J. Physiol.*, 1933, CV, 191–216) have discovered a new hormone, prolactin, which is secreted by the anterior lobe of the pituitary gland, travels in the blood stream, and causes the secretion of milk.

J. S. H.

**Occurrence of Metals in the Liver.**—H. RAMAGE, J. H. SHELDON, AND WILFRID SHELDON (*Proc. Royal Soc.* [London], Series B, 1933, CXIII, 308–327) have subjected the livers of 125 children to spectrographic analysis. At birth, the liver contains a reserve supply of iron and copper; other metallic elements, such as manganese, rubidium, and calcium, are also present. The iron reserve is increased during the first two months of life. While the child is on a milk diet, the iron, copper, manganese, and calcium concentrations in the liver decrease, while the concentration of potassium and of rubidium increases. After the child is placed on a mixed diet, the content of the liver in iron, copper, manganese, and calcium increases. Approximately one-third of the livers contained silver.

HENRY GEORGE REES (*Analyst*, 1933, LVIII, 384–387) analyzed fresh ox liver and found 0.0080 per cent of iron and 0.0024 per cent. of copper. On extraction with water at a temperature of 50° C., 38.1 per cent. of the total iron and 7.3 per cent. of the total copper passed into the aqueous extract. Application of a higher temperature was less effective for extraction of the iron.

J. S. H.

**Lactic Acid Content of Milk Products.**—In the course of a paper on the quantitative determination of lactic acid in milk and its products, by oxidation to oxalic acid, and precipitation of calcium oxalate, B. G. HARTMANN AND F. HILLIG (*Jour. Asso. Official Agric. Chem.*, 1933, XVI, 435–445) report the per cent. of lactic acid found in certain dairy products: Fluid whole milk (4 samples) 0.025 to 0.029, whole milk powder (3 samples) 0.217 to 0.275, evaporated milk (3 samples) 0.057 to 0.084, sweetened condensed milk (1 sample) 0.125; milk fresh from cow 0.022, after 24 hours at room temperature 0.057, signs of curdling 0.133, curdled 0.287.

J. S. H.

**Influence of Sulphur Compounds on Growth.**—DOROTHY WALL HAMMETT AND FREDERICK S. HAMMETT (*Protoplasma*, 1933, XIX, 161–169) have demonstrated that growth by increase in cell number is regulated by the naturally occurring chemical equilibrium composed of the sulphydryl group and its partially oxidized derivatives. For instance, when the chela or claw of the hermit crab is removed, it regenerates by growth by increase in cell number. This growth is accelerated by organic compounds containing sulphydryl, e.g., by para thio cresol, and is retarded by partially oxidized organic compounds of sulphur, e.g., by diphenyl sulphoxide.

J. S. H.

**Bromides from the Sea.**—(*Ind. & Eng. Chem.*, News Edition, Vol. 11, p. 244.) A real effort will be made to recover bromine from sea water. The process involves many problems in efficient chemical engineering. The concentration of bromine in the sea is one pound per 2,000 gallons of water, a condition which up to now has discouraged commercial attempts upon its recovery. However, plenty of bromine is to be had, for in 1 cubic mile of sea water will be found 600 million pounds of bromine and there is no scarcity of cubic miles of sea water.

The recovery plant, installed by well-known American chemical manufacturing firms, is intended to produce at least 500,000 pounds of bromine a month. To do this, 26,000 gallons of sea water per minute must flow through the plant for 24 hours a day. It requires less than sixty seconds for the water to be processed during which time it undergoes chemical treatment, loses its small trace of bromine and is on its way back to the sea in a thoroughly debrominated condition. The process consists of treating the sea water with enough chlorine to decompose the bromides in the water. The bromine thus liberated is blown out of the sea water by a current of air. The air containing the bromine is then brought into contact with soda solution which absorbs the element, forming a concentrated solution of sodium bromide and bromate.

This bromine recovery plant is to be located at Kure Beach about 20 miles south of Wilmington, N. C. The location is characterized by an abundance of sea water, relatively clean, not diluted with surface water, containing relatively little organic matter and a convenience in disposal of the waste water so it does not mix with the intake.

C.

**Vitamin G.**—This vitamin is water-soluble, promotes growth, and, according to some authorities, prevents pellagra. LELA E.

BOOHER (*Jour. Biol. Chem.*, 1933, CII, 39–46) has obtained from whey powder a product in which vitamin G has a concentration 40 to 50 fold that in the original powder, and 2,000 times that in fresh whole milk, as determined by experiments on rats. This vitamin is probably identical with the water-soluble, yellow-green fluorescent pigment of whey known as lactochrome.

J. S. H.

**Mesocystine.**—HUBERT S. LORING AND VICENT DU VIGNEAUD (*Jour. Biol. Chem.*, 1933, CII, 287–295) describe mesocystine, the stereoisomer of the amino acid cystine which is optically inactive by internal compensation.

J. S. H.

**Pharmacology of Thallium.**—JAMES C. MUNCH AND JAMES SILVER (U. S. Dept. Agric. Technical Bull. No. 238, 1931, 28 pages) have conducted an extensive research on the pharmacology of thallium which was introduced in 1920 as a poison for rodents. When fed to either wild or albino rats as the sulphate, the minimum lethal dose is 25 milligrams per kilogram of body weight; death usually occurs on the second or third day, and is due to respiratory failure. On intravenous injection into rabbits, the minimum lethal dose is also 25 milligrams per kilogram of body weight. In poisoning by thallium, the chief symptoms are loss of hair, pains in the muscles and nerves of the legs, disturbances in the glands of internal secretion, and a disturbed calcium metabolism which gives rise to rickets.

Thallium is among the most toxic substances used for rat control. It has about the same toxicity as strychnine, is four times as toxic as arsenious oxide, ten times as toxic as red squill powder, and thirty times as toxic as barium carbonate. These figures are based on the respective minimum lethal doses as determined by feeding experiments.

"Thallium is a cumulative poison of high toxicity and is without taste, smell, or other warning property. It should not be recommended to the public as a rodent poison. Where the use of thallium is found necessary for the control of highly resistant species of rodents, it should be entrusted only to persons who understand its dangerous qualities and who will exercise appropriate care in handling it."

J. S. H.

**Potato Poisoning.**—S. G. WILLIMOTT (*Analyst*, 1933, LVIII, 431–438) describes an outbreak of solanine poisoning in Cyprus in

1932 due to the use, by the natives, of potato shoots and leaves as a green vegetable. The shoots and leaves were gathered at about the time of flowering, cooked by boiling, and eaten. Approximately 50 cases of poisoning occurred; the symptoms were essentially those of a gastro-enteritis. In one family, 8 persons (ages from 4 to 52 years) developed symptoms approximately 12 hours after eating a meal which contained young potato shoots; one patient died in 44 hours; the others were definitely improved in 4 days, and eventually recovered completely. The food ingested was free from inorganic poisons, but the potato shoots contained the gluco-alkaloid, solanine. On analysis, the solanine content of potatoes (4 samples) was found to be between 2.7 and 9.1 mg. per 100 grams of sample, while the solanine content of potato shoots was between 27.0 and 48.9 mg. per 100 grams of sample. The solanine content tended to decrease with an increase in the altitude at which the potatoes and their shoots were grown. Albino rats, although omnivorous feeders, refused to eat potato shoots. Warning against the danger of using potato plants as food led to a discontinuance of the practice, and terminated the occurrence of this form of poisoning.

J. S. H.

**Selenium from the Nutritional Aspect.**—Selenium has recently been found in several foods of vegetable origin. TABOURY (*Comptes Rendus*, 1932, CXCV, 171) analyzed several species of plants from La Roche-Posay. The mineral water of that place contains selenium; and these plants were growing in the bed or on the wet banks of a stream fed by that water. He found selenium in both the parsnip and the water parsnip. W. O. ROBINSON (*Journal of the Association of Official Agricultural Chemists*, 1933, XVI, 423–424) has found selenium in some samples of natural wheat. A toxic wheat contained 10 to 12 parts of selenium per million, while its gluten contained 90 parts of selenium per million. Another sample of wheat contained 5 to 6 parts of selenium per million, while samples of other non-toxic wheat were free from selenium.

E. M. NELSON, A. M. HURD-KARRER, and W. O. ROBINSON (*Science*, 1933, LXXVIII, 124) report the development of a pronounced toxicosis or poisoning when toxic wheat or its straw is fed to rats or guinea pigs. Growth is retarded; and death occurs in a few weeks. Wheat, which had absorbed selenium from the soil and contained 8 to 10 parts of that element per million, frequently produced readily detectable macroscopic changes in the livers of the experimental animals. Danger exists in the use of selenium insecticides. Complete removal of the spray residue from the

sprayed fruit or vegetable will not remove any toxic selenium compounds which have formed within the tissues of the plant.

J. S. H.

**Magnesium Pyrophosphate.**—One gravimetric procedure for the determination of phosphorus and its compounds depends upon the precipitation of magnesium ammonium phosphate. This compound is ignited to obtain magnesium pyrophosphate which is weighed. SAMUEL J. KIEHL and HENRY B. HARDT (*Jour. Am. Chem. Soc.*, 1933, LV, 3555–3560) have studied certain changes which occur during the ignition. The flash or luminescence is due to liberation of heat and increase in temperature as the amorphous pyrophosphate crystallizes. The precipitate is a hexahydrate. As this water of hydration is lost during the early stages of ignition, certain gaseous organic compounds, if present, become firmly attached to the phosphate, and later break down and leave a residue of graphitic carbon. This phenomenon explains the dark residue which remains in the crucible, instead of the white residue which is expected. The organic compounds may be supplied by charring filter paper. The monohydrate of magnesium ammonium phosphate does not possess this property of adsorption of gaseous carbon compounds to any appreciable extent.

J. S. H.

**Anhydrous Acetic Acid.**—WILLIAM C. EICHELBERGER AND VICTOR K. LA MER (*Jour. Am. Chem. Soc.*, 1933, LV, 3633–3634) find that the most satisfactory method for the preparation of pure, anhydrous acetic acid is the removal of impurities by oxidation with chromium trioxide, followed by dehydration with triacetyl borate. The pure, anhydrous acid melted at $16.60° \pm 0.01°$ C., and had a specific conductance of $1.4 \times 10^{-8}$ mhos at a temperature of 25° C.

J. S. H.

**Calgon—a New Agent for the Improvement of Laundering.**—It has been found in a joint investigation of the Buromin Company of Pittsburgh, Mellon Institute, and the American Institute of Laundering that Calgon, a new phosphate, possesses distinct utility in laundering processes.

According to the report on this research, Calgon has the unique property of dissolving lime soaps by transforming them into active sodium soaps. The utilization of this property in the washing formula is said to represent a real innovation in laundering methods, because it enables the removal of lime soaps as an integral part of

the washing formula. The occurrence of lime soaps in soft-water laundries is pointed to as far more widespread than is commonly thought. It was proved that Calgon is not injurious in any way to textile fabrics nor to colors, and that its use effects an economy in washing supplies.

R.

**Heavy Soil Erosion.**—(U. S. Department of Agriculture, Clip Sheet No. 798.) Unless the land is terraced, continuous cropping of cotton in the highly erosive soil which makes up approximately 36,000,000 acres in Oklahoma and Texas will result in wearing all the top soil away in 30 years and leave the stiff clay subsoil which produces only about half as much, the work of the United States Department of Agriculture soil-erosion experiment station at Guthrie, Okla., indicates.

The experiments at this station, known as the Red Plains station, show that unless some grain or grass crop is grown the soil loss amounts to 32.5 tons an acre a year and that 14 per cent. of the rainfall is lost by run-off. Where grass was grown on similar slopes, the soil loss was only 60 pounds to the acre and only 1.7 per cent. of the rain was lost.

Of about 16,000,000 acres of this erosive type of soil in cultivation in Oklahoma, about 13,000,000 are suffering seriously from erosion and about 7,000,000 of these have reached the gullying stage. In the last 10 years 1,500,000 acres have been ruined by deep washing and gullying.

To combat this erosion the Department in urging more grass crops, terracing, and soil-saving dams to stop gullying, and similar practices.

R.

**The Tulip Tree Comes Back.**—(U. S. Dept. of Agric. Clip Sheet. No. 794.) According to the United States Forest Service the tulip tree, also known as the yellow poplar, is replacing the blight-killed chestnut on some of the best sites in the Appalachian region. The tulip tree will not take root and grow indiscriminately but seems to require spaces that are fairly open; where the soil is deep; well supplied with moisture, yet well drained. Natural regeneration by wind-blown seed, from seed trees at least 35 years old, appears to be the most effective way in which the tulip tree reproduces itself. The tree will not thrive well in close competition with other vigorous hardwoods and so should be planted in open spots.

Lightness of the wood, making it easy to float down the streams, led to heavy cutting and resulted in early elimination of yellow poplar from many of the best bottom lands. The easy workability

of the wood has also contributed to its popularity as a source of lumber. The large attractive blossoms of the tree form a good source of honey. A well-developed tree of 20 years will yield enough nectar to bees to make 4 pounds of this natural confection.

C.

**The World's Deepest Well.**—(White Metal News Letter, Sept. 1933.) In the great Kettleman Hills oil field of Southern California veteran drilling crews have penetrated the earth's surface to a depth of more than two miles, the greatest ever attained. All this drilling is being done in an effort to reach the Eocene rocks which have been found quite productive in other California fields. The well was started April 12, 1932 and a depth of over 11,000 feet already has been reached, exceeding the previous record by 415 feet.

The initial drilling was made with a 25-inch bit and a 24-inch reamer made of nickel alloy steel. As the depth increased, smaller sizes of these reamers were substituted, until at the time the record depth was achieved a $5\frac{3}{8}$-inch was being used.

C.

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# AWARDS BY THE INSTITUTE

The following awards are made by The Franklin Institute:

**The Franklin Medal** (Gold Medal).—This medal is awarded annually from the Franklin Medal Fund, founded January 1, 1914, by Samuel Insull, Esq., to those workers in physical science or technology, without regard to country, whose efforts, in the opinion of the Institute, acting through its Committee on Science and the Arts, have done most to advance a knowledge of physical science or its applications.

**The Elliott Cresson Medal** (Gold Medal).—This medal is awarded for discovery or original research, adding to the sum of human knowledge, irrespective of commercial value; leading and practical utilizations of discovery; and invention, methods or products embodying substantial elements of leadership in their respective classes, or unusual skill or perfection in workmanship.

**The Howard N. Potts Medal** (Gold Medal).—This medal is awarded for distinguished work in science or the arts; important development of previous basic discoveries; inventions or products of superior excellence or utilizing important principles.

**The George R. Henderson Medal** (Gold Medal).—This medal is to be awarded for meritorious inventions or discoveries in the field of railway engineering.

**The John Price Wetherill Medal** (Silver Medal).—This medal is awarded for discovery or invention in the physical sciences or for new and important combinations of principles or methods already known.

**The Edward Longstreth Medal** (Silver Medal).—This medal is awarded for inventions of high order and for particularly meritorious improvements and developments in machines and mechanical processes. In the event of an accumulation of the fund for medals beyond the sum of one hundred dollars, it is competent for the Committee on Science and the Arts to offer from such surplus a money premium for some special work on any mechanical or scientific subject that is considered of sufficient importance.

**The Louis E. Levy Medal** (Gold Medal).—This medal is awarded to the author of a paper of especial merit, published in the JOURNAL OF THE FRANKLIN INSTITUTE, preference being given to one describing the author's experimental and theoretical researches in a subject of fundamental importance.

**The Walton Clark Medal** (Gold Medal).—This medal is awarded to the "author of the most notable advance in knowledge or improvement in apparatus, or in method concerning the science or the art of gas manufacture or distribution or utilization in the production of illumination, or of heat, or of power."

**The Certificate of Merit.**—A Certificate of Merit is awarded to persons adjudged worthy thereof for meritorious inventions, discoveries or improvements in physical processes or devices.

**The Boyden Premium.**—Uriah A. Boyden, Esq., of Boston, Mass., has deposited with The Franklin Institute the sum of one thousand dollars, to be awarded as premium to "any resident of North America who shall determine by experiment whether all rays of light and other physical rays are or are not transmitted with the same velocity."

**For further information relating to these awards apply to the Secretary of the Institute**